Do Short Soccer Players Have an Advantage Over Tall Players?

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Why are some soccer players able to move more quickly around the field than others? What attributes of a player come into play? Mental alertness, muscle action, fitness, body mass, and height are all likely candidates. And a taller soccer player has an obvious advantage when the ball is airborne.

But what about when the ball is on the ground? Would a shorter player be able to accelerate or to change running direction more quickly than a taller player, thereby offering the shorter player an occassional competitive advantage?

The analysis presented here addresses these questions by analysing the prevailing dynamics both in the brief period before a player takes the first step to begin running, and in the ensuing period of acceleration. The analysis will show that for two players of equal mass, the shorter player has an advantage over the taller player, and that *the advantage is determined by the square root of the ratio of their respective heights.*

Simple pendulum idealisation. Suppose a player wishes to start running in the direction $\hat{\mathbf{x}}$. To do so, the player must orient his body in such a way that he is able to exert a force in that direction. But because the human body has predominantly vertical extent with limited lateral extent (in contrast to a dog's body, say), the player must first lean into the direction $\hat{\mathbf{x}}$ before he may exert such a force.

This is shown schematically in Figure 1. The player is idealised by assuming that his body mass is concentrated at the centre of mass m located a distance l from his feet on the ground. Under such an idealisation, only two forces act on the player's body, namely, his weight $\mathbf{W}$ directed downwards, and a reaction force $\mathbf{R}$ which he applies in a radial direction.

Choosing for the sake of convenience to use polar coordinates (r, θ) instead of the usual cartesian coordinates (x, y), the player's centre of mass is located at the vector position $\mathbf{r} = l\hat{\mathbf{r}}$. To analyse the motion of m under the influence of the forces, we must express acceleration in polar coordinates as well, taking note that unlike the cartesian basis vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$, the polar basis vectors $\hat{\mathbf{r}}$ and $\hat{\theta}$ are not stationary. Their directions change as the mass moves along its circular path. That is $\hat{\mathbf{r}} = \hat{\mathbf{r}}(t)$ and $\hat{\theta} = \hat{\theta}(t)$.

In polar coordinates, the velocity is

$$\dot{\mathbf{r}} \;=\; l\frac{\mathrm{d}\hat{\mathbf{r}}}{\mathrm{d}t}$$

But what is $\frac{\mathrm{d}\hat{\mathbf{r}}}{\mathrm{d}t}$? Over an infinitesimal time increment $\mathrm{d}t$, the vector $\hat{\mathbf{r}}$ changes from $\hat{\mathbf{r}}(t)$ to $\hat{\mathbf{r}}(t+\mathrm{d}t)$. And the difference vector is $\mathrm{d}\hat{\mathbf{r}}(t) = \hat{\mathbf{r}}(t+\mathrm{d}t) - \hat{\mathbf{r}}(t)$, as shown schematically in Figure 2. It points in the direction $\hat{\theta}$.

That is

$$\begin{aligned}
\mathrm{d}\hat{\mathbf{r}}(t) &= \hat{\mathbf{r}}(t+\mathrm{d}t) - \hat{\mathbf{r}}(t)\\
&= |\,\mathrm{d}\hat{\mathbf{r}}(t)|\hat{\theta}\\
&= |\hat{\mathbf{r}}|\,\mathrm{d}\theta\,\hat{\theta}\\
&= \mathrm{d}\theta\,\hat{\theta}
\end{aligned}$$

The velocity vector is therefore

$$\dot{\mathbf{r}} \;=\; l\dot{\theta}\,\hat{\theta}$$

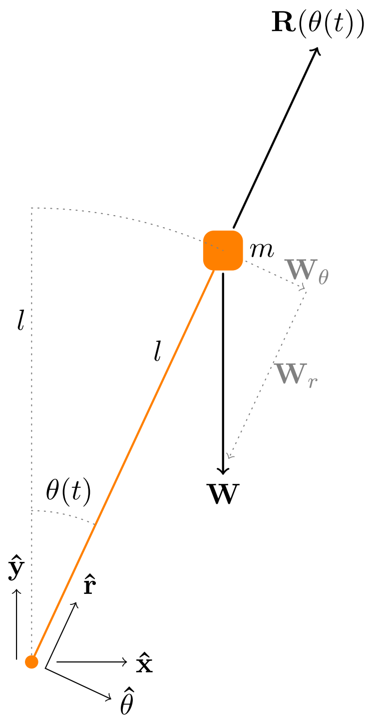


Figure 1: The simple pendulum idealisation.

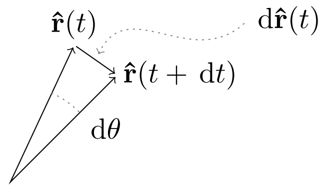


Figure 2: Elemental difference vector $\mathrm{d}\hat{\mathbf{r}}(t)$.

In polar coordinates, the acceleration is

$$\begin{aligned}\ddot{\mathbf{r}} &= \frac{\mathrm{d}\dot{\mathbf{r}}}{\mathrm{d}t} \\ &= \frac{\mathrm{d}}{\mathrm{d}t}(l\dot{\theta}\,\hat{\theta}) \\ &= l\ddot{\theta}\hat{\theta} \;+\; l\dot{\theta}\frac{\mathrm{d}\hat{\theta}}{\mathrm{d}t}\end{aligned}$$

Applying the same graphical reasoning to $\mathrm{d}\hat{\theta}(t)$, as shown in Figure 3, we observe that

$$\begin{aligned}\mathrm{d}\hat{\theta}(t) &= \hat{\theta}(t+\mathrm{d}t) - \hat{\theta}(t) \\ &= |\,\mathrm{d}\hat{\theta}(t)|(-\hat{\mathbf{r}}) \\ &= |\hat{\theta}|\,\mathrm{d}\theta\,(-\hat{\mathbf{r}}) \\ &= -\,\mathrm{d}\theta\,\hat{\mathbf{r}}\end{aligned}$$

The acceleration vector is therefore

$$\ddot{\mathbf{r}} \;=\; -l\dot{\theta}^2\,\hat{\mathbf{r}} \;+\; l\ddot{\theta}\,\hat{\theta} \tag{1}$$

Initial "falling" period. While the soccer player allows his body to "fall" down the circular arc (Figure 1), he increases his leaning angle from the initial angle θ_0 to the final angle $\theta(t)$. This

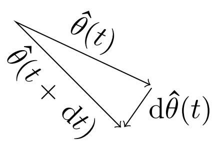


Figure 3: Elemental difference vector $\mathrm{d}\hat{\theta}(t)$.

initial falling period is to prepare himself for a subsequent period of sustained acceleration. During this early transient time, his body is subjected to two external two forces, namely, $\mathbf{R}$ and $\mathbf{W}$. The resultant force is the vector sum of the two:

$$m\frac{\mathrm{d}^2\mathbf{r}(t)}{\mathrm{d}t^2} = \mathbf{R}(\theta(t)) + \mathbf{W} \tag{2}$$

I emphasise that during this initial period, his centre of mass is following a downward circular arc trajectory, and $\mathbf{R}$ is simply his reactive force which he must apply in order to maintain this trajectory. Nothing more. And since $\mathbf{R}$ is perpendicular to the motion of his centre of mass, the player does no work during this period. However, as soon as the required leaning angle $\theta(t)$ is obtained, the player immediately enters a period of sustained acceleration in the direction $\hat{\mathbf{x}}$, during which time the leaning angle is maintained, and his centre of mass ceases to move along the circular arc, but instead simply moves along $\hat{\mathbf{x}}$. In this latter period, he will need to exert an additional force—a force which does work, consuming energy.

For now, we concern ourselves with the dynamics of the initial falling period. In polar coordinates, $\mathbf{R}$ has only a radial component, while $\mathbf{W}$ has both a radial and an angular component (Figure 1):

$$\begin{aligned} \mathbf{R}(\theta) &= R(\theta)\,\hat{\mathbf{r}} \\ \mathbf{W} &= W_r\,\hat{\mathbf{r}} + W_\theta\,\hat{\theta} = -W\cos\theta\,\hat{\mathbf{r}} + W\sin\theta\,\hat{\theta} \end{aligned} \tag{3}$$

Combining (1), (2) and(3) gives

$$-l\dot{\theta}^2\,\hat{\mathbf{r}} + l\ddot{\theta}\,\hat{\theta} = R(\theta)\,\hat{\mathbf{r}} - W\cos\theta\,\hat{\mathbf{r}} + W\sin\theta\,\hat{\theta}$$

from which two differential equations are obtained:

$$\boxed{\begin{aligned} & l\ddot{\theta} - W\sin\theta = 0 \\ & R(\theta) + l\dot{\theta}^2 - W\cos\theta = 0 \end{aligned}} \tag{4}$$

“Falling” to leaning angle. A solution to the first equation will offer a prescription for the change in angle θ over time once the player has started to lean. Conversely, if we are able to invert that prescription, then we will know how long it will take for the player to lean into $\hat{\mathbf{x}}$ by some specified angle θ.

If we are able to find a solution $\theta = \theta(t)$, then we are permitted to state $\dot{\theta}(t) = \dot{\theta}(\theta(t))$, so that using the Chain Rule of the differential calculus

$$\ddot{\theta}(t) = \ddot{\theta}(\theta(t)) = \frac{\mathrm{d}\dot{\theta}(\theta(t))}{\mathrm{d}t} = \frac{\mathrm{d}\dot{\theta}(\theta)}{\mathrm{d}\theta}\,\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} = \dot{\theta}\,\frac{\mathrm{d}\dot{\theta}(\theta)}{\mathrm{d}\theta}$$

The first equation in (4) may then be expressed as

$$\dot{\theta}\,\frac{\mathrm{d}\dot{\theta}}{\mathrm{d}\theta} = \frac{W}{l}\sin\theta$$

Integrating

$$\int \dot{\theta}\,\mathrm{d}\dot{\theta} \;=\; \frac{W}{l}\int \sin\theta\,\mathrm{d}\theta$$

so that

$$\dot{\theta}^2 = -\frac{2W}{l}\cos\theta + C \qquad \text{for some integration constant } C$$

It is reasonable to assume that the player starts leaning from a stationary position inclined at an initial angle θ_0, giving $C = (2W/l)\cos\theta_0$, so that the solution to the player's time rate of change of angle is therefore

$$\dot{\theta}(\theta(t)) \;=\; \sqrt{\frac{2W}{l}}\sqrt{\cos(\theta_0) - \cos(\theta(t))} \tag{5}$$

To solve for $\theta(t)$, we must integrate (5)

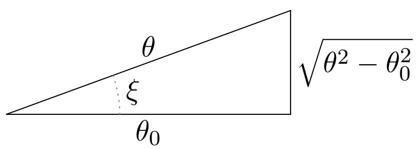
$$\int_{\theta_0}^{\theta} \frac{1}{\sqrt{\cos\theta_0 - \cos\phi}}\,\mathrm{d}\phi \;=\; \sqrt{\frac{2W}{l}}\,t \tag{6}$$

The integral in (6) is difficult to work with. However, since both θ and θ_0 are small, the cos functions may be approximated using the first two terms of their Taylor expansions: $\cos x \approx 1 - \frac{1}{2}x^2$ for small x, resulting in

$$\int_{\theta_0}^{\theta} \frac{1}{\sqrt{\phi^2 - \theta_0^2}}\,\mathrm{d}\phi \;=\; \sqrt{\frac{W}{l}}\,t$$

It is easy to integrate this by applying the change of variable

$$\begin{aligned} \theta(\xi) &= \theta_0 \sec\xi \\ \mathrm{d}\theta(\xi) &= \theta_0 \sec\xi \tan\xi\,\mathrm{d}\xi \end{aligned}$$



so that

$$\begin{aligned} \sqrt{\frac{W}{l}}\,t &= \int_{\theta_0}^{\theta} \frac{1}{\theta_0}\frac{1}{\sqrt{\sec^2\xi - 1}}\,\theta_0\sec\xi\tan\xi\,\mathrm{d}\xi \\ &= \int_{\theta_0}^{\theta} \sec\xi\,\mathrm{d}\xi \\ &= \ln(\sec\xi + \tan\xi) \end{aligned}$$

Therefore, finally

$$\boxed{t(\theta) \;=\; \sqrt{\frac{l}{W}}\,\ln\left(\frac{\theta + \sqrt{\theta^2 - \theta_0^2}}{\theta_0}\right)} \tag{7}$$

This is the prescription for how long it will take for the soccer player to lean forward by angle θ, assuming that he begins from a stationary position at angle θ_0, and provided that θ and θ_0 are small. The result confirms our intuition that as the player leans, his sweep of angle becomes progressively larger through time.

In a plot of this relationship between θ and t shown in Figure 4, it is assumed the player's height is 1.75m, his centre of mass is located at 0.6×1.75m from the ground up, his mass is 75kg so that his weight is 75×9.8N, and that he is initially stationary leaning at an angle 0.1°.

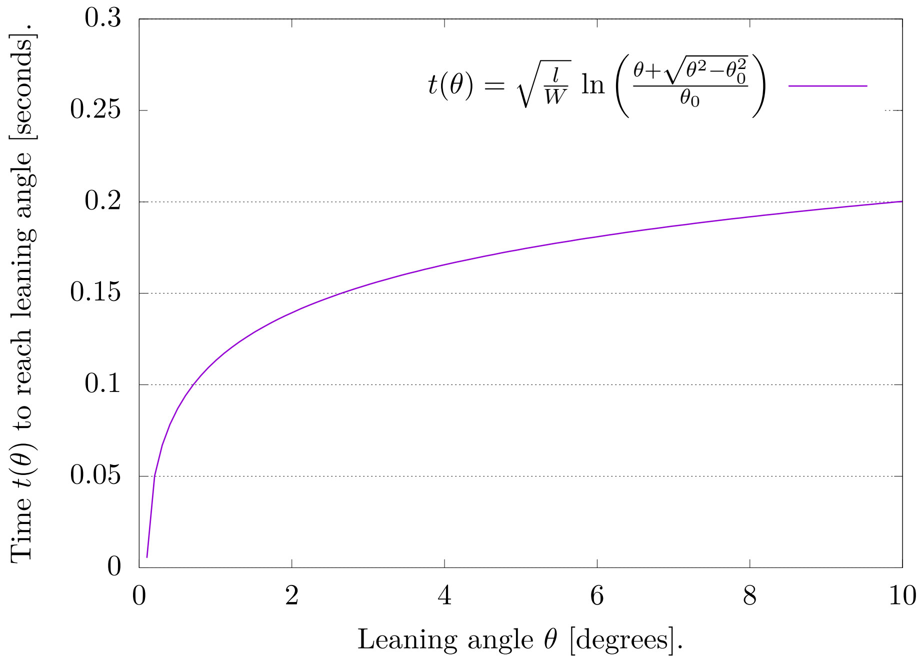


Figure 4: Time taken for a soccer player to lean forward to subtend an angle θ following a stationary position at initial angle $\theta_0 = 0.1°$. The height of the centre of mass is l=0.6×1.75m. The mass of the player is m=75kg, so that his weight is W=75×9.8N.

The result (7) hints at the possibility that a shorter player has an advantage over a taller player during the initial falling period. The time to reach the required leaning angle increases with the square root of the height l of the player's centre of mass. However, we cannot yet be sure of this advantage because we do not yet know what the target leaning angle is. It might be that players of different heights require different leaning angles. This will be addressed later.

Reactive force while "falling". By substituting (5) into (4), the player's reactive force during the initial falling period is calculated as

$$\mathbf{R}(\theta) \;=\; W(3\cos\theta - 2\cos\theta_0)\,\hat{\mathbf{r}}$$

This confirms our intuition that if $\theta = \theta_0 = 0$, then $\mathbf{R} = W\,\hat{\mathbf{r}}$. Also, it is interesting to observe that at an angle $\arccos(\frac{2}{3}\cos\theta_0)$, which is approximately equal to $\arccos(\frac{2}{3}) = 48.1°$ for small θ_0, $\mathbf{R}(\theta)$ reverses direction, pointing along $-\hat{\mathbf{r}}$.

Subsequent accelerating period. While dribbling with the ball, chasing after the ball, or tackling the opposition, the player's ability to accelerate rapidly from a standing position is advantageous. This acceleration occurs in a period subsequent to the initial falling period, as discussed above.

Suppose that after leaning, the player moves in the direction $\hat{\mathbf{x}}$ with constant acceleration a. The leaning angle now remains constant, and the vertical component of the movement of his body vanishes, leaving a component along $\hat{\mathbf{x}}$. However, to sustain this period, the player will need to exert an additional force—a force which does work, consuming energy. And as with $\mathbf{R}$, the only direction in which he may exert this force is along $\hat{\mathbf{r}}$. But since his leaning angle θ is now non-neglible (albeit most likely still small), such exertion $\mathbf{E} = E\,\hat{\mathbf{r}}$ has a required component along $\hat{\mathbf{x}}$. The equation of motion is now

$$m\frac{\mathrm{d}^2\mathbf{r}(t)}{\mathrm{d}t^2} \;=\; ma\,\hat{\mathbf{x}} \;=\; \mathbf{R} + \mathbf{W} + \mathbf{E}$$

And using (3)

$$ma\,\hat{\mathbf{x}} \;=\; R\,\hat{\mathbf{r}} \;+\; W(-\hat{\mathbf{y}}) \;+\; E\,\hat{\mathbf{r}}$$

With reference to Figure 1 we have

$$\begin{aligned} \hat{\mathbf{r}} &= \sin\theta\,\hat{\mathbf{x}} \;+\; \cos\theta\,\hat{\mathbf{y}} \\ \hat{\theta} &= \cos\theta\,\hat{\mathbf{x}} \;-\; \sin\theta\,\hat{\mathbf{y}} \end{aligned}$$

giving

$$ma\,\hat{\mathbf{x}} \;=\; (R+E)\sin\theta\,\hat{\mathbf{x}} + (R+E)\cos\theta\,\hat{\mathbf{y}} - W\,\hat{\mathbf{y}}$$

from which

$$ma \;=\; (R+E)\sin\theta \qquad \text{and} \qquad (R+E)\cos\theta - W = 0$$

so that

$$a = \frac{W}{m}\tan\theta = g\tan\theta$$

$$\text{or} \quad \boxed{\theta = \arctan(a/g)}$$

Competitive advantage. It is thus apparent that the target leaning angle required for a player to move with a specified constant acceleration does not depend on his intrinsic attributes, such as his height and mass, but only on how quickly he wishes to get going. With this knowledge, we may now complete the analysis by updating (7):

$$\boxed{t \;=\; t(l,m,a) \;=\; \sqrt{\frac{l}{mg}}\,\ln\left(\frac{\arctan(a/g) + \sqrt{\arctan^2(a/g) - \theta_0^2}}{\theta_0}\right)} \tag{8}$$

Consider two soccer players. One is short, with his centre of mass m_S at height l_S. The other is tall, with his centre of mass m_T at height l_T. Both players wish to move under a constant acceleration a. To do so both players will need to lean forward by the same angle $\theta = \arctan(a/g)$ starting from an initial small angle θ_0. But their respective times taken to reach that leaning angle will differ, given by (8). The ratio of their two times is

$$\boxed{\frac{t_T}{t_S} \;=\; \frac{t(l_T, m_T, a)}{t(l_S, m_S, a)} \;=\; \sqrt{\frac{l_T}{l_S}\cdot\frac{m_S}{m_T}}}$$