A Rooftop Solar Power Analysis

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Abstract

In a country with a faltering supply of baseload electricity provided by one or more large central power utilities, alternative supplies of electricity necessarily become attractive. One such alternative is photovoltaic solar power. And indeed, as an alternative, decentralised rooftop solar power installations are more and more becoming a visual feature across South Africa's urban landscape.

Unfortunately, rooftop solar power installations suffer high up-front costs. So to assist in the efficient allocation of these costs, this study helps to specify the required "size" of a rooftop installation. In particular, based on a physical model of solar insolation, this study specifies both the required number of solar panels and number of batteries comprising an installation. In the physical model, a novel concept dubbed the "Effective Global Horizontal Irradiation" is introduced to tractably account for the effect of tilting the solar panels towards the sun.

Contents

1 Glossary

1. PV—Photovoltaic.
2. Inverter—A physical electronic device that converts DC current into AC current. Solar panels and batteries output DC current. But electrical appliances input AC current. Hence the need for an inverter.
3. Charge controller—A voltage and/or current regulator device to keep batteries from overcharging. I think it is part of a hybrid inverter.
4. DoD—Depth of Discharge is that maximum acceptable amount of battery energy discharge expressed as a fraction of the full charge. A DoD not exceeding 80% for lithium batteries will prolong their life.
5. Solar Irradiance—The solar power received per unit area (W/m^2).
6. DNI—Direct Normal Irradiance or beam radiation (W/m^2) is the solar irradiance per unit area perpendicular to the beam line. It is measured at the surface of the Earth. The DNI is equal to the extraterrestrial direct normal irradiance above the atmosphere minus power losses due to atmosperhic absorption and scattering. Herein it is denoted I_{N}.
7. DHI—Diffuse Horizontal Irradiance (W/m^2) is the radiant power per unit horizontal area at the Earth's surface obtained from sunlight which was first scattered by the atmosphere. Therefore, were there to be no atmosphere, DHI would be zero. Herein it is denoted I_{D}.
8. GHI—Global Horizontal Irradiance (W/m^2) is the total solar power received per unit horizontal area at the Earth's surface. Herein it is denoted I_{GH}.
9. GHR—Global Horizontal Irradiation (J/m^2 or Wh/m^2) is the total solar energy received per unit horizontal area at the Earth's surface in some specified time interval $[t_0, t]$. Typical intervals are one hour, one day or one year. The GHR is often called the insolation or the solar flux density. Herein it is denoted R_{GH}. So
$$R_{\mathrm{GH}}(t; t_0) = \int_{t_0}^{t} I_{\mathrm{GH}}(\theta(\tau))\,\mathrm{d}\tau$$
10. GTI—Global Tilted Irradiance (W/m^2) is the total solar power received per unit area at the Earth's surface, but where the surface area is not horizontal but is oriented with a tilt angle and azimuthal angle.
11. STC—Standard Test Conditions is a set of agreed conditions under which the electrical power output of solar panels are tested. The conditions are:[1]
    - Irradiance of 1000 W/m^2.
    - Panel temperature of 25°C.
    - Air mass fraction of 1.5.

    Panel temperatures are usually higher than ambient temperature. The air mass fraction is the mass of atmospheric air relative to the mass of atmospheric air at the equator. Europe's air mass fraction is about 1.5.
12. NOC—Normal Operating Conditions is the set of common reference conditions designed to simulate panel performance for actual outdoor measurements. It is reasonable to assume that the panel power output at NOC is 80% of power output at STC.[1]
13. Peak Watts—A solar panel's power output measured under STC.[1] It is often denoted as Wp.
14. PSH—Peak Sun Hour. One peak sun hour *is defined* as the amount of time required for an irradiance of 1000 W/m^2 to transfer to a perpendicular surface an amount of radiant energy equal to 1000 Wh/m^2. Obviously this time is exactly 1 hour.
15. MPP—Maximum Power Point is a solar panel's output voltage at which the panel's electrical power production under STC is maximised.

16. MPPT—Maximum Power Point Tracker is an electronic DC-to-DC converter that optimizes the match between the solar array (PV panels) and the battery bank. It converts a higher DC voltage output from a solar panel array to the lower DC voltage needed to charge batteries. It enables the maximum available electrical power to be extracted from a solar panel array. It is included in charge controllers.
17. BMS—Basic Management System.
18. CoC—Certificate of Compliance.
19. AGM—Absorbed Glass Mat. AGM batteries contain special glass mats which are located inside the battery between the battery plates. The glass mats wick the electrolyte solution so that the solution is not stored in a free liquid form. AGM batteries are more resistant to chemical corrosion, and so have a longer lifespan than standard lead acid batteries.

2 Simple model

A typical rooftop solar energy installation is shown in Figure 1. The solar panel array, the hybrid inverter, and the battery bank are all installed on the premises of the household. When planning an installation, two important questions are:

1. How many solar panels are required?
2. How many batteries are required?

To help answer these questions, energy balances at the solar panel array and at the batteries need to be coupled to the electrical energy demands of the household.

The amount of electrical energy that a household is able to obtain from a solar panel array over a given time interval depends on many factors, such as, the intensity of the incident solar radiation, the total surface area exposure to this radiation by the array, the time of exposure, and the operational efficiency of the panels, the inverter and the batteries. As the sun's radiant energy impinges on the solar panels, a fraction of this energy will be converted to useful electrical energy. The remaining energy will be lost either as reflected short wave radiation or as long wave heat. Our interest here obviously lies with the useful fraction.

Solar panels are typically tilted at an angle relative to the horizontal. This is to try to maximise the solar incident energy flux as the sun traces its trajectory across the sky. To begin the analysis, however, I shall assume that the panels are installed horizontally. Under this assumption, we may easily appeal to the incident Global Horizontal Irradiance (W/m^2) from the sun. Then later, I relax the assumption by accounting for the effect of tilting the solar panels.

The solar panels

Global Horizontal Irradiance and Irradiation. The sun traces a predictable but non-trivial path across the sky during the day. This means that the solar energy received by the sun at any given location on Earth also varies non-trivially throughout the day. It varies as a function of the sun's zenith angle and whether or not there are clouds in the sky. The zenith angle is shown in Figure 2 as $\theta(t)$.

In characterising energy received from the sun, two important concepts come up. The first is the concept of the Global Horizontal Irradiance, I_{GH}. And the second is the Global Horizontal Irradiation, R_{GH}. Despite the resemblance of the two words, *irradiance* and *irradiation*, they mean quite different things. Irradiance is to *power* as irradiation is to *energy*.[2] Irradiance concerns the instantaneous rate of solar irradiant energy transfer over some area. So its units are watts per square meter (W/m^2). Irradiation concerns the cumulative transfer of solar radiant energy over some area for a specified time period (e.gs., hourly, daily, monthly or annually). Its common units are kilowatt-hours per square meter (kWh/m^2). For these units to be interpreted correctly, however, the time interval must be stated (e.gs., kWh/m^2 daily, or kWh/m^2 annually).

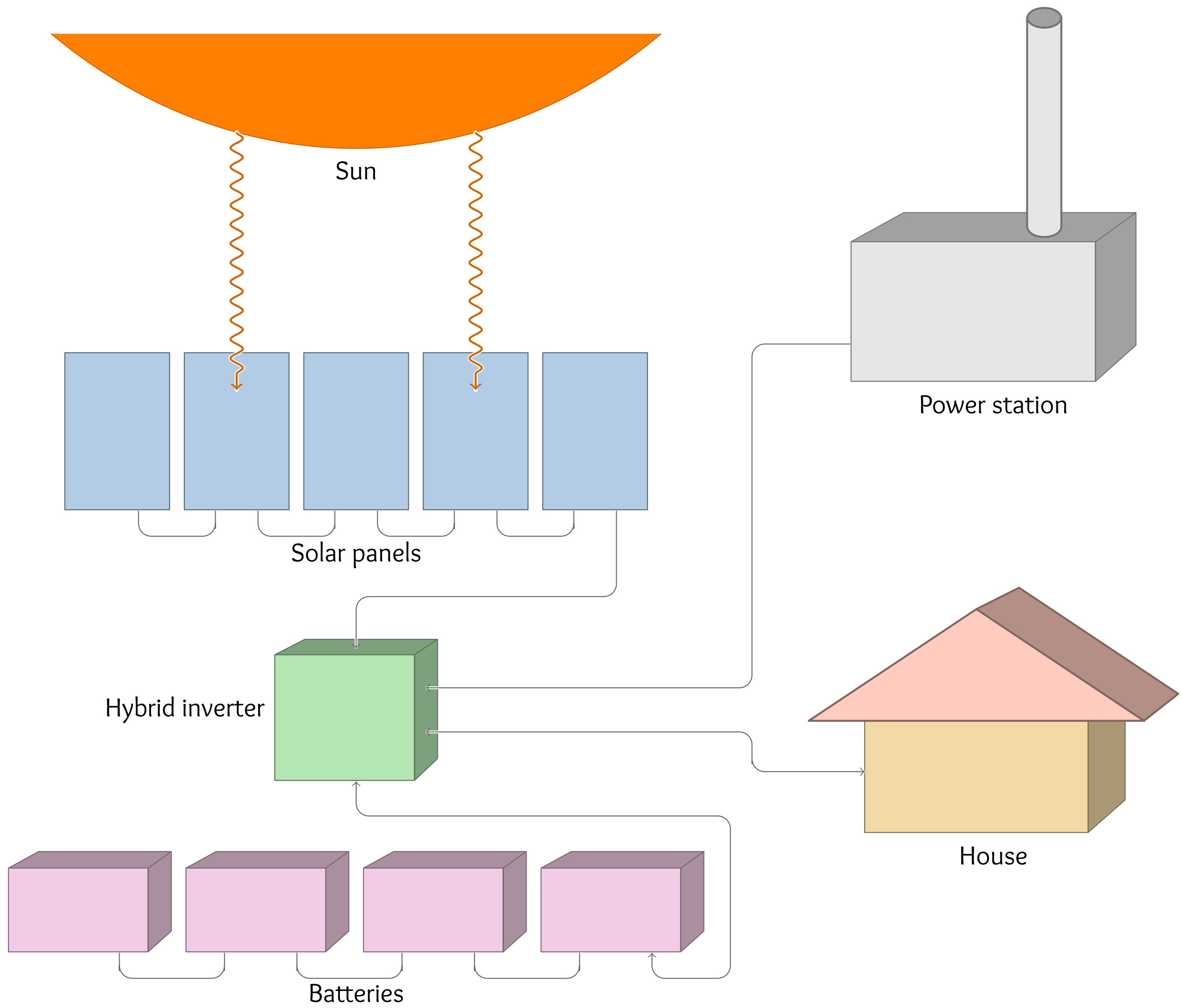

Figure 1: Typical rooftop solar energy installation.

The Global Horizontal Irradiance, $I_{\mathsf{GH}}(t)$, is defined as the irradiant solar energy received per unit time per unit horizontal surface area located on the Earth's surface. An element $\mathrm{d}A$ of such a horizontal surface area is shown in Figure 2. The Global Horizontal Irradiance may be partitioned as the sum of the incident power per unit horizontal area obtained directly from the sun and the incident power per unit horizontal area obtained diffusely. That is

$$I_{\mathsf{GH}}(t) = I_{\mathsf{N}} \cos\theta(t) + I_{\mathsf{D}}(t) \tag{1}$$

where I_{N} is the Direct Normal Irradiance. As its names suggests, I_{N} is the solar radiant energy received per unit time per unit surface area *oriented perpendicularly* to the incident sun rays. $I_{\mathsf{D}}(t)$ is the Diffuse Horizontal Irradiance. It is the instantaneous radiant energy received per unit time per unit horizontal area, but *not* received directly from the sun, instead received diffusely. And $\theta(t)$ is the instantaneous solar zenith angle as a function of some time t. The units of I_{GH}, I_{N} and I_{D} are W/m^2.

If the total surface area of a solar panel array is A, say, then by definition of $I_{\mathsf{GH}}(t)$, the total radiant energy incident on the array over some time interval $[t_0, t_1]$ must be

$$E_{\mathsf{GH}} = \int_A \int_{t_0}^{t_1} I_{\mathsf{GH}}(t)\,\mathrm{d}t\mathrm{d}S = A \int_{t_0}^{t_1} I_{\mathsf{GH}}(t)\,\mathrm{d}t \tag{2}$$

Since household electrical energy consumption cycles daily, we are interested in the total radiant energy incident on the solar panel array over a full day. To reflect this interest, we may set the $[t_0, t_1]$ time interval

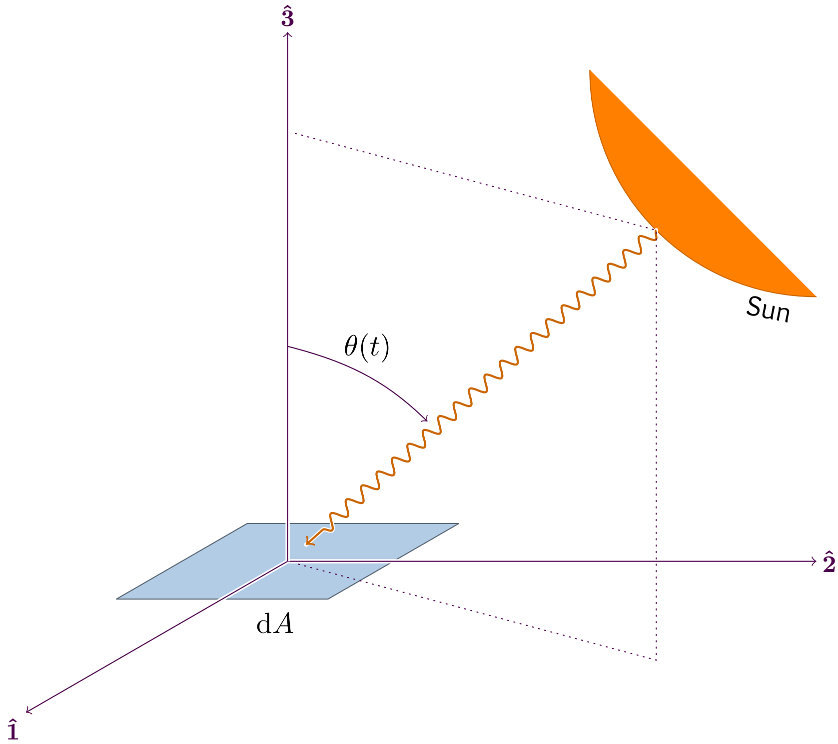

Figure 2: Solar zenith angle $\theta(t)$ subtended by the sun's rays incident upon a horizontally oriented surface element $\mathrm{d}A$.

to span a full day:

$$\begin{aligned}
E_{\text{GH}} &= A\left[\int_{\text{day}} + \int_{\text{night}}\right] I_{\text{GH}}(t)\,\mathrm{d}t \\
&= A\int_{\text{day}} I_{\text{GH}}(t)\,\mathrm{d}t \;+\; 0 \\
&= A\Delta t\left[\frac{1}{\Delta t}\int_{\text{day}} I_{\text{GH}}(t)\,\mathrm{d}t\right] \\
&= A\Delta t\,\langle I_{\text{GH}}\rangle
\end{aligned}$$

where $\langle I_{\text{GH}}\rangle$ is by definition the average of $I_{\text{GH}}(t)$ over the daytime part of a day. The quantity $\Delta t\langle I_{\text{GH}}\rangle$ is obviously the total energy received per unit area during daytime. It can be calculated by integrating $I_{\text{GH}}(t)$. But doing so is not necessary because the energy received per unit area in a day has been measured for different places around the world. Thus we happily write

$$E_{\text{GH}} = A\Delta t\,\langle I_{\text{GH}}\rangle = AR_{\text{GH}} \tag{3}$$

where R_{GH} is the measured Global Horizontal Irradiation received over daytime hours for a particular surface location of interest on Earth. Its common units are kilowatt-hours per square meter (kWh/m^2). Note here that as discussed on page 3, 'irradiation' is not the same as 'irradiance'. A distribution map of values for R_{GH} for South Africa is shown in Figure 5 on page 11.

Unfortunately, not all of the incident energy, E_{GH}, each day is converted to useful electrical energy. The conversion is limited by the panels' photovoltaic efficiency, ϵ. But if this efficiency is known, then we may assert that the total electrical energy produced by the panels in a day must be ϵE_{GH}, and so we write

$$E_{\text{S}} = \epsilon E_{\text{GH}} = \epsilon AR_{\text{GH}} \tag{4}$$

Trends in photovoltaic conversion efficiencies are shown in Figure 6 on page 12.

Panel power rating and Standard Test Conditions. The photovoltaic efficiency ϵ for commercial panels is not normally reported. What is instead reported is the electrical power output of a panel under so-called Standard Test Conditions (STC). Under STC, the incident Global Horizontal Irradiance, I_{GH}, is fixed at

$$\langle I_{\text{GH}}\rangle_{\text{STC}} = 1000 \text{ W/m}^2$$

Suppose that under STC, the electrical power output of a single panel is measured to be p_{STC}, say. And if the area of a single panel is known to be A_1, then the panel's photovoltaic efficiency at STC must by its definition be

$$\epsilon_{\text{STC}} = \frac{p_{\text{STC}}}{\langle I_{\text{GH}}\rangle_{\text{STC}} A_1} \tag{5}$$

We now assume that the panel photovoltaic efficiency is constant, so that

$$\epsilon = \epsilon_{\text{STC}} \tag{6}$$

Under this assumption, substituting (5) and (6) into (4) gives

$$E_{\text{S}} = \frac{p_{\text{STC}} R_{\text{GH}}}{\langle I_{\text{GH}}\rangle_{\text{STC}}} \frac{A}{A_1} = \frac{p_{\text{STC}} R_{\text{GH}}}{\langle I_{\text{GH}}\rangle_{\text{STC}}} N_{\text{S}} \tag{7}$$

where $N_{\text{S}} \equiv A/A_1$ is the number of panels in the solar array.

The household

For the electrical energy E_{S} produced by the solar panel array to be of any use, the energy must flow through the inverter, as shown in Figure 1 on page 4. The inverter works to convert the incoming direct current (DC) electricity into useful alternating current (AC) electricity as required by the household appliances. Unfortunately, inverters are not perfectly efficient. Between 5% and 15% of the energy entering the inverter will be lost as heat. That is, the operational efficiency, η, of an inverter is between 85% and 95%.[3] And so, the useful available energy leaving the inverter obtained from the sun for a day is

$$E_{\text{I}} = \eta E_{\text{S}} = \eta \frac{p_{\text{STC}} R_{\text{GH}}}{\langle I_{\text{GH}}\rangle_{\text{STC}}} N_{\text{S}} \qquad \text{(From (7))} \tag{8}$$

Let E_{H} be the total electrical energy demand by the household for a typical 24-hour day. Let h be the fraction of energy required by the household during the time of the day when the solar panels are receiving Global Horizontal Irradiation (Eq. (2)). That is, during the daytime part of the day. Then the household energy demand for the full 24-hour day may be partitioned as

$$E_{\text{H}} = hE_{\text{H}} + (1-h)E_{\text{H}} \tag{9}$$

Now let s be the fraction of household energy which we would like to obtain from the solar panel array each full 24-hour day. Then $(1-s)$ is the fraction of household energy which we must obtain from somewhere else, either from an external baseload utility (Figure 1) or from a generator. Of course, being wholly independent of an external electricity energy supplier means to set $s = 1$. For the solar panel array to supply sE_{H} of the household's energy demand, we require to set

$$E_{\text{I}} = sE_{\text{H}} = shE_{\text{H}} + s(1-h)E_{\text{H}} \tag{10}$$

The energy partition in (9) was carried over into (10) merely to show that the energy originating from the solar panel array and passing through the inverter exceeds shE_{H}, which is the household's energy demand only while the sun is shining. Combining (8) with (10) gives

$$N_{\text{S}} = \frac{s}{\eta} \frac{\langle I_{\text{GH}}\rangle_{\text{STC}}}{p_{\text{STC}} R_{\text{GH}}} E_{\text{H}} \tag{11}$$

Clearly, the number of solar panels required increases with increase in s and E_{H}, and decreases with increase in η, p_{STC} and R_{GH}, as expected.

The batteries

The batteries must energise the household when solar energy is not available. The life of many lithium ion and lithium iron phosphate ($LiFePO_4$) batteries may be prolonged if they are never fully discharged during use. Let d be the maximum depth-of-discharge fraction. Typically, $d = 80\%$. Let E_B be the total energy capacity of the N_B batteries, and e_B be the energy capacity of a single battery. Then clearly

$$E_B = N_B e_B \tag{12}$$

Obviously, the energy demand by the household at night must be supplied by the batteries via the inverter. That is, the batteries must do for the inverter at night what the solar panels are doing for the inverter during the day. So we must set

$$\eta(dE_B) = s(1-h)E_H$$

Substituting (12) into this gives

$$\boxed{N_B = \frac{s(1-h)}{\eta d}\,\frac{E_H}{e_B}} \tag{13}$$

This informs that the number of batteries required increases with increase in s and E_H, and decreases with increase in d, h and e_B, as expected.

Tilting the solar panels

It has been assumed thus far that the solar panels are oriented horizontally on a household's roof. The assumption enabled easy recourse to the measured radiant quantities of Global Horizontal Irradiance I_{GH} (Eq. (1)) and Global Horizontal Irradiation R_{GH} (Eq. (3)), with the latter aggregated over a typical day. By measuring R_{GH} over a day, for any particular location on Earth, the influence of the sun's non-trivial trajectory across the sky is aggregated. In particular, for high latitude locations, the sun follows a trajectory which lies closer to that location's horizon. And so that location's measured R_{GH} values will typically be smaller in aggregate than for lower latitude locations.

But while the measured values of R_{GH} account for the effect of the sun's trajectory, they do not account for the effect of irradiated surfaces on Earth not being oriented horizontally. And in practice, solar panels are usually not oriented horizontally but are tilted towards the sun. Tilting in this way serves to increase the *effective surface area* of the panel upon which the sun's rays are incident.

Effective irradiated surface area. Consider a typical solar panel shown in Figure 3. The panel is tilted towards the sun by a *tilt angle* β. The optimal year-round solar panel tilt angle for Johannesburg, South Africa, is usually set to $\beta = 25.1°$ from the horizontal.[4] I currently live in Johannesburg.

The sun's maximum elevation angle in mid-winter is α, say. The panel's surface area is $A = uv$. Because the panel is tilted towards the sun, the effective surface area of the panel increases.

To calculate the effective surface area, note that

$$\begin{aligned}\frac{z}{u} &= \cos\beta\\ \frac{y}{u} &= \sin\beta\\ \frac{y}{x} &= \tan\alpha\end{aligned}$$

From these equations it is easy to obtain

$$\begin{aligned}x + z &= \frac{u\sin\beta}{\tan\alpha} + u\cos\beta\\ &= \left[\frac{\sin\beta\cos\alpha + \sin\alpha\cos\beta}{\sin\alpha}\right]u\\ &= \frac{\sin(\alpha+\beta)}{\sin\alpha}\,u\end{aligned}$$

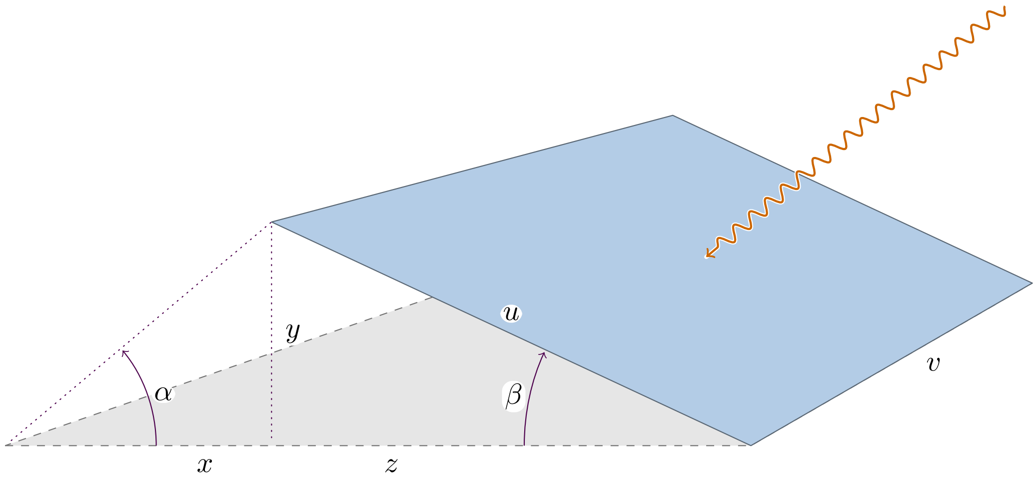

Figure 3: Solar panel tilted towards the sun by a tilt angle β. The mid-winter sun's midday elevation angle relative to the horizontal is denoted α. The tilt increases the effective surface area of the solar panel. The optimal year-round solar panel tilt angle for Johannesburg, South Africa, is usually set to $\beta = 25.1°$C from the horizontal.

And so the effective surface area of the tilted panel is

$$A'(\alpha, \beta) = (x + z)v = \frac{\sin(\alpha + \beta)}{\sin \alpha} A \tag{14}$$

To confirm this result, $A'(\alpha, \beta)$ is maximised when

$$\frac{\partial A'(\alpha, \beta)}{\partial \beta} = \frac{\cos(\alpha + \beta)}{\sin \alpha} A$$

vanishes. That is, when $\beta = \dfrac{\pi}{2} - \alpha = 90° - \alpha$, as expected.

Solar elevation angle. During winter in Johannesburg, South Africa, the angular position of the sun as it moves across the sky during the day from east to west is closer to the horizontal plane than it is during summer. That is, the angle between the northern horizontal plane and the sun's rays is smaller in winter than it is in summer. The angle is called the *elevation angle* and is denoted α. We are interested in the mid-winter angle because its smallness limits where on our property we can place an array of solar panels. The maximum elevation angle occurs at noon. The diagram in Figure 4 shows the geometrical arrangement of the Earth and the sun at noon in the middle of winter in Johannesburg, South Africa.

How do we calculate this elevation angle α? The diagram shows a cross-section of the Earth and the sun as the Earth follows its annual orbital trajectory around the sun. As the Earth moves, it will be moving into the page. But the Earth also spins around an axis which is tilted relative to a virtual line perpendicular to the Earth's orbital plane. The axis and the virtual line are shown in Figure (4) as dotted lines. The extent of the tilt is known and is measured by the so-called *declination angle*, $\delta = 23.5°$, as shown in the diagram. Also, Johannesburg is located in the southern hemisphere at a line of latitude which subtends an angle $\phi = 26.2°$ relative to the equator. This is also shown in the diagram.

It is easy to see that

$$\begin{aligned} \delta + \psi + \phi &= 90° \\ \psi + \gamma &= 90° \\ \gamma + 90° + \alpha &= 180° \end{aligned}$$

From which it is easy to solve for α as

$$\alpha = 90° - \delta - \phi = 40.3° \tag{15}$$

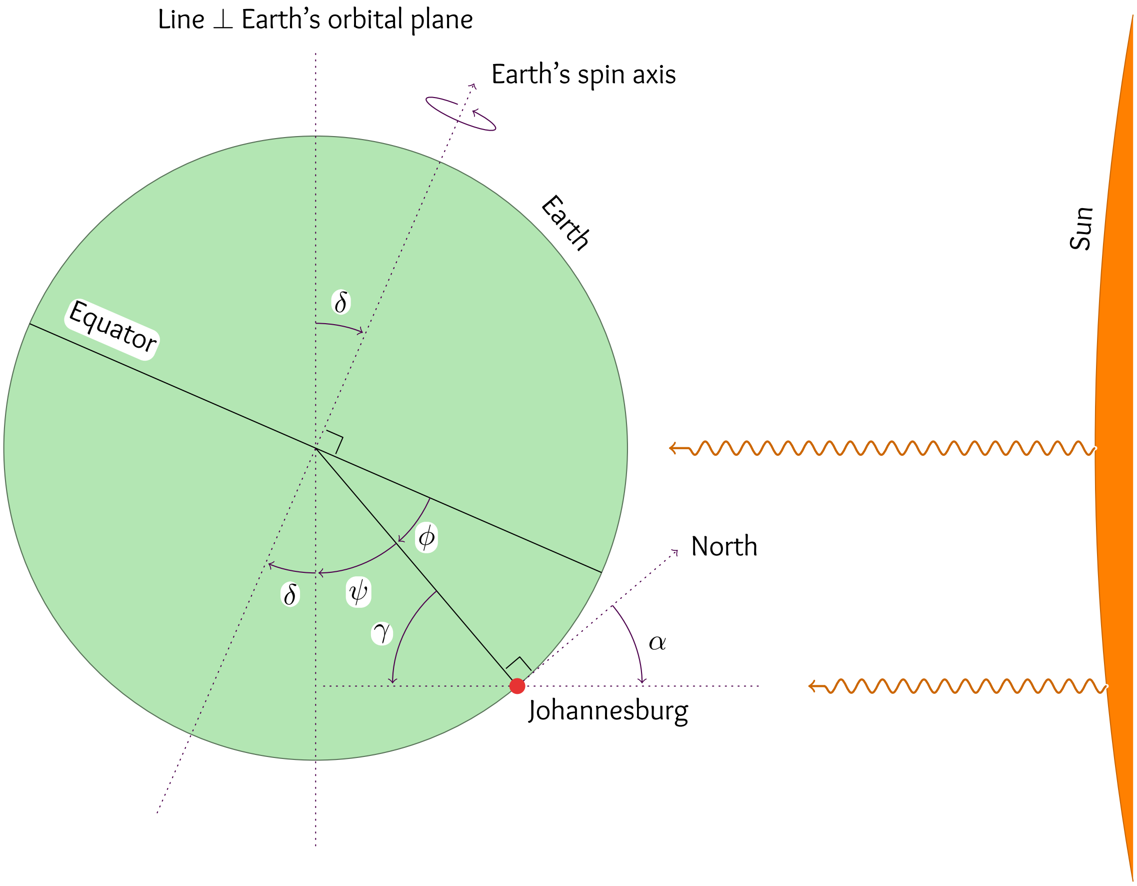

Figure 4: Geometrical arrangement for the calculation of the maximum solar elevation angle α at noon in Johannesburg, South Africa, in the middle of winter. The orbital plane lies perpendicular to the page. The declination angle is $\delta = 23.5°$. Johannesburg's angle of latitude is $\phi = 26.2°$. The elevation angle is calculated here to be $\alpha = 40.3°$.

Effective Global Horizontal Irradiation. It is reasonable to assume that the increased surface area $A'(\alpha, \beta)$ in (14) and as shown in Figure 3 on page 8 which obtains when the solar panels are tilted towards the sun effectively increases only the *direct* irradiation from the sun's rays, with little to no effect on the *diffuse* irradiation. So how do we account for this tilting effect? From (2), the total radiant energy incident on the array over some time interval $[t_0, t_1]$ is

$$\begin{aligned} E_{\mathsf{GH}} &= A \int_{t_0}^{t_1} I_{\mathsf{GH}}(t)\,\mathrm{d}t \\ &= A \int_{t_0}^{t_1} \big(I_{\mathsf{N}} \cos\theta(t) + I_{\mathsf{D}}(t)\big)\,\mathrm{d}t \qquad \text{(Using (1))} \qquad (16) \\ &= A I_{\mathsf{N}} \int_{t_0}^{t_1} \cos\theta(t)\,\mathrm{d}t + A \int_{t_0}^{t_1} I_{\mathsf{D}}(t)\,\mathrm{d}t \end{aligned}$$

By tilting the solar panels towards the sun by angle β, the effective incident radiant energy becomes, using (14)

$$\begin{aligned} E'_{\mathsf{GH}} &= \frac{\sin(\alpha+\beta)}{\sin\alpha} A I_{\mathsf{N}} \int_{t_0}^{t_1} \cos\theta(t)\,\mathrm{d}t \;+\; A \int_{t_0}^{t_1} I_{\mathsf{D}}(t)\,\mathrm{d}t \\ &= \frac{\sin(\alpha+\beta)}{\sin\alpha} \left(E_{\mathsf{GH}} - A \int_{t_0}^{t_1} I_{\mathsf{D}}(t)\,\mathrm{d}t\right) \;+\; A \int_{t_0}^{t_1} I_{\mathsf{D}}(t)\,\mathrm{d}t \qquad \text{(Using (16))} \qquad (17) \\ &= \frac{\sin(\alpha+\beta)}{\sin\alpha} E_{\mathsf{GH}} \;-\; \left(\frac{\sin(\alpha+\beta)}{\sin\alpha} - 1\right) A \int_{t_0}^{t_1} I_{\mathsf{D}}(t)\,\mathrm{d}t \end{aligned}$$

Computing E'_{GH} over an arbitrary $[t_0, t_1]$ time interval requires the integral of $I_{\mathsf{D}}(t)$ to be evaluated. But as with R_{GH} in (3), since we are interested in a time interval spanning a single day, we may make recourse to measured values as follows:

$$
\begin{aligned}
\int_{t_0}^{t_1} I_\mathsf{D}(t)\,\mathrm{d}t &= \left[\int_\text{day} + \int_\text{night}\right] I_\mathsf{D}(t)\,\mathrm{d}t \\
&= \int_\text{day} I_\mathsf{D}(t)\,\mathrm{d}t \,+\, 0 \\
&= \Delta t \left[\frac{1}{\Delta t}\int_\text{day} I_\mathsf{D}(t)\,\mathrm{d}t\right] \\
&= \Delta t\,\langle I_\mathsf{D}\rangle
\end{aligned}
$$

where $\langle I_\mathsf{D}\rangle$ is by definition the average of $I_\mathsf{D}(t)$ over the daytime part of a day. The quantity $R_\mathsf{D} \equiv \Delta t\langle I_\mathsf{D}\rangle$ is called the Diffuse Horizontal Irradiation. It is obviously the total radiant energy received *diffusely* per unit horizontal surface area during daytime. It has been measured for different places around the world.

So using (3), the Effective Global Horizontal Irradiation may be written as:

$$
E'_\mathsf{GH} = \frac{\sin(\alpha+\beta)}{\sin\alpha}\,AR_\mathsf{GH} - \left(\frac{\sin(\alpha+\beta)}{\sin\alpha} - 1\right) AR_\mathsf{D}
$$

Following (4) and using (5) and (6), the total electrical energy produced by the tilted panel in a day is adjusted to

$$
E'_\mathsf{S} = \epsilon E'_\mathsf{GH} = \left[\frac{\sin(\alpha+\beta)}{\sin\alpha}R_\mathsf{GH} - \left(\frac{\sin(\alpha+\beta)}{\sin\alpha} - 1\right) R_\mathsf{D}\right] \frac{p_\mathsf{STC}}{\langle I_\mathsf{GH}\rangle_\mathsf{STC}}\,N_\mathsf{S} \tag{18}
$$

where as in (7), $N_\mathsf{S} \equiv A/A_1$ is the number of panels in the solar array.

Following (8), the adjusted useful available energy leaving the inverter for a day due to tilted panels is

$$
E'_\mathsf{I} = \eta E'_\mathsf{S} \tag{19}
$$

And following (10), for an array of tilted panels to supply sE_H of the household's energy needs, we require to set

$$
E'_\mathsf{I} = sE_\mathsf{H}
$$

Using (18) and (19) and solving for N_S gives the required number of solar panels as

$$
\boxed{N_\mathsf{S} \equiv \frac{A}{A'} = \frac{s}{\eta}\,\frac{\sin\alpha}{\sin(\alpha+\beta)R_\mathsf{GH} - \big(\sin(\alpha+\beta) - \sin\alpha\big)R_\mathsf{D}}\,\frac{\langle I_\mathsf{GH}\rangle_\mathsf{STC}}{p_\mathsf{STC}}\,E_\mathsf{H}} \tag{20}
$$

As expected, this expression for N_S reduces to (11) when the effect of solar panel tilting is removed by setting $\beta = 0$.

Battery charging time. Suppose we are given a single 12 V, 100 Ah battery. And suppose that the string of solar panels offers a current into the battery of 20 A. How long will it take to charge the battery?

Firstly, we need to establish what 100 Ah is.

Electrical current is defined as the amount of electrical charge flowing in an electrical circuit per second. That is, $1\,\mathrm{A} = 1\,\mathrm{C/s}$. Therefore

$$
100\,\mathrm{Ah} = 100\,\mathrm{C/s} \times 3600\,\mathrm{s} = 360\,000\,\mathrm{C}
$$

This shows that 100 Ah is a measure of electrical charge stored in the battery. Since the solar panels offer an electrical current of 20 A, it means that the panels offer 20 C/s into the battery. This implies that the panels will take

$$
\frac{360\,000\,\mathrm{C}}{20\,\mathrm{C/s}} = \frac{100\,\mathrm{Ah}}{20\,\mathrm{A}} = 5\,\mathrm{h}
$$

to charge the battery.

3 Solar data

The Global Solar Atlas. The Energy Sector Management Assistance Program (ESMAP) is a multi-donor trust fund administered by the World Bank. The fund supports a global initiative known as Renewable Energy Resource Mapping. The initiative focuses on biomass, small hydro, solar, and wind data. The solar part of this initiative includes the Global Solar Atlas project to provide online access to solar data globally.[5][6] The distribution map for Global Horizontal Irradiation, R_{GH}, for South Africa was downloaded from the Global Solar Atlas's website and is shown in Figure 5. Specific per day values of $R_{\mathrm{GH}} = 5467$ Wh/m^2 and $R_{\mathrm{D}} = 1759$ Wh/m^2 for Johannesburg, South Africa, were obtained from an interactive map on the website.

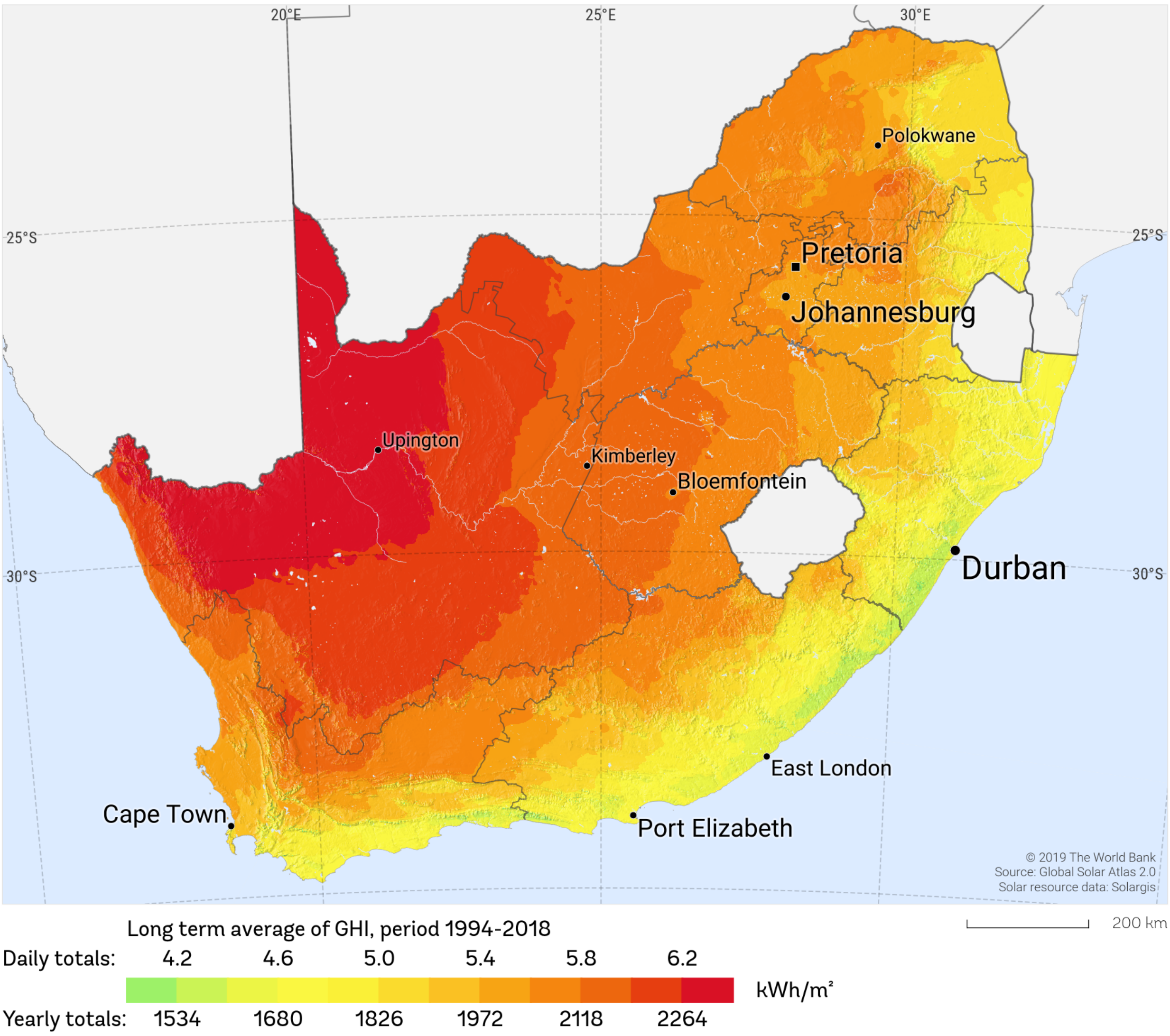

Figure 5: Long-term average of the Global Horizontal Irradiation, R_{GH}, for South Africa.[7][5]

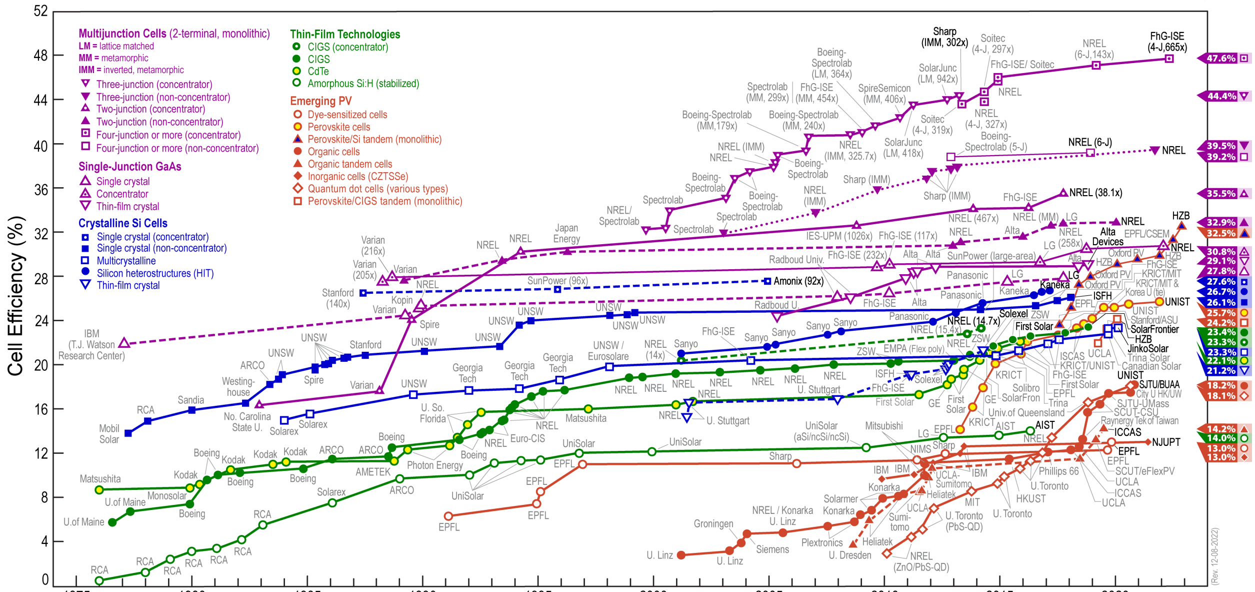

Figure 6: Solar panel cell energy conversion highest efficiencies of solar photovoltaic cells worldwide from 1976 through 2022 for various photovoltaic technologies. This data is tracked by the US Department of Energy's National Renewable Energy Laboratory (NREL).[8][9] The efficiencies of commercial solar panels will likely be lower.

4 Our home energy usage

I recorded our household's electrical energy consumption as follows:

Date	*Time*	*Hours*	*Meter Reading* [kWh]	*Energy Use* [kWh]
20Jan23	03h40		620.51	
21Jan23	04h50	25.167	610.19	10.32
23Jan23	04h30	47.667	579.36	30.83
24Jan23	09h45	29.25	563.80	15.56
25Jan23	06h45	21.0	553.39	10.41
26Jan23	03h50	21.083	539.14	14.25
27Jan23	04h30	24.667	520.35	18.79
3Feb23	09h45	173.25	409.22	111.13
9Feb23	22h30	156.75	304.03	105.19
16Feb23	20h08	165.633	202.86	101.17
		Total: 664.467		*Total*: 417.65

The average electrical energy consumed per unit time is therefore 417.65 kWh/664.467 h=628.548 W. This means that the total electrical energy demand for a typical day is

$$E_{\mathsf{H}} = 628.548\ \mathsf{W} \times 24\,\mathsf{h} = 15085.173\ \mathsf{Wh} \tag{21}$$

5 Results

The objective in this work was to better understand how radiant energy may be harnessed from the sun and converted into household electrical energy via a rooftop solar installation. This improved understanding may now be used to help answer the two important questions posed earlier. Namely:

1. How many solar panels are required?
2. How many batteries are required?

Number of solar panels. Using the above physical model, from (20) an expressions for the number of required solar panels is

$$N_{\mathsf{S}} = \frac{s}{\eta}\,\frac{\sin\alpha}{\sin(\alpha+\beta)R_{\mathsf{GH}} - \big(\sin(\alpha+\beta)-\sin\alpha\big)R_{\mathsf{D}}}\,\frac{\langle I_{\mathsf{GH}}\rangle_{\mathsf{STC}}}{p_{\mathsf{STC}}}\,E_{\mathsf{H}}$$

where

s is the fraction of household energy which we would like to obtain from the solar panel array each day (cf (10)). So to model the effect of drawing no electrical energy from an external baseload utility, we must set $s = 1$.

η is the operational efficiency of the inverter (cf (8)). Its value lies between 85% and 95%. So for now, we set $\eta = 85\% = 0.85$.

α is the maximum solar elevation angle at noon in mid-winter relative to the horizontal plane (cf Figure 3 on page 8, Figure 4 on page 9, and equation (15)). For Johannesburg, South Africa, its value is $40.3°$.

β is the angle by which the solar panels are tilted towards the sun (cf Figure 3) at noon in mid-winter. For Johannesburg, we set its value to $25.1°$.

$\langle I_{\mathsf{GH}}\rangle_{\mathsf{STC}}$ is the incident Global Horizontal Irradiance, I_{GH}, but under Standard Test Conditions (STC). The STC fixes its value at exactly $1000\,\mathsf{W/m^2}$ (cf (2)).

R_{GH} is the Global Horizontal Irradiation received during daytime of a typical day. Using data from the Global Solar Atlas project (Figure 5), for Johannesburg, we set its value at 5467 Wh/m^2.

R_{D} is the Diffuse Horizontal Irradiation received during the daytime of a typical day. Using data from the Global Solar Atlas project, for Johannesburg, we set its value at 1759 Wh/m^2.

p_{STC} is the electrical power output of a single solar panel under STC. Typical values for rooftop solar panels are in the range 450 W to 550 W. For now, I shall use 455 W.

E_{H} is the total household electrical energy demand for a typical day. Our household's demand was measured to be (cf (21)) 15085.173 Wh.

Substituting these data into the above expression gives the number of required solar panels for our household as

$$\begin{aligned}
N_{\mathsf{S}} &= \frac{1}{0.85}\,\frac{\sin(40.3^\circ)}{\sin(40.3^\circ+25.1^\circ)\times 5467\;-\;\big(\sin(40.3^\circ+25.1^\circ)-\sin(40.3^\circ)\big)\times 1759}\,\frac{1000}{455}\,E_{\mathsf{H}} \\
&= 3.708\times 10^{-4}\,E_{\mathsf{H}} \qquad\qquad \text{in units W, Wh, Wh/m}^2 \\
&= 5.59
\end{aligned}$$

Number of batteries. Using the above model, from (13), an expressions for the number of required batteries is

$$N_{\mathsf{B}} = \frac{s(1-h)}{\eta d}\,\frac{1}{e_{\mathsf{B}}}\,E_{\mathsf{H}}$$

where

$s = 1$, as above.

$\eta = 0.85$, as above.

h is the fraction of energy required by the household during the time of the day when the solar panels are receiving Global Horizontal Irradiation from the sun (cf (9)). Since a typical household consumes about 60% of its daily energy needs while the sun is *not* shining, and 40% while the sun is shining, we set $h = 0.4$.

d is advised maximum battery depth-of-discharge fraction in order to prolong the life of lithium ion and lithium iron phosphate (LiFePO$_4$) batteries. Typically, $d = 0.8$, but here we set conservatively $d = 0.7$.

e_{B} is the energy capacity of a single battery. A typical lithium ion battery has an energy capacity of about $e_{\mathsf{B}} = 5100$ Wh.

$E_{\mathsf{H}} = 15085.173$ Wh, as above.

Substituting these data into the above expression gives the number of required batteries for our household as

$$\begin{aligned}
N_{\mathsf{B}} &= \frac{1(1-0.4)}{0.85\times 0.7}\,\frac{1}{5100}\,E_{\mathsf{H}} \\
&= 1.977\times 10^{-4}\,E_{\mathsf{H}} \qquad\qquad \text{in units W, Wh, Wh/m}^2 \\
&= 2.98
\end{aligned}$$

References

[1] Alternative Energy Tuturials. Standard Test Conditions. Retrieved from https://www.alternative-energy-tutorials.com/photovoltaics/standard-test-conditions.html. Accessed 1 February 2023.

[2] PV Performance Modelling Collaborative. Irradiance & Insolation. Retrieved from https://pvpmc.sandia.gov/modeling-steps/1-weather-design-inputs/irradiance-and-insolation-2/. Accessed 24 March 2023.

[3] Linquip. Efficiency of Inverter: Calculation and Equation Formula Guide. Retrieved from https://www.linquip.com/blog/efficiency-of-inverter/. Accessed 16 February 2023.

[4] Footprint Hero. Solar Panel Tilt Angle Calculator. Retrieved from https://footprinthero.com/solar-panel-tilt-angle-calculator. Accessed 27 January 2023.

[5] Global Solar Atlas. Global Horizontal Irradiation for South Africa. Retrieved from https://globalsolaratlas.info/download/south-africa. Accessed 9 February 2023.

[6] Wikipedia contributors. Global Solar Atlas. Retrieved from https://en.wikipedia.org/wiki/Global_Solar_Atlas. Accessed 24 March 2023.

[7] Solargis. Solar resource maps of South Africa. Retrieved from https://solargis.com/maps-and-gis-data/download/south-africa. Accessed 9 February 2023.

[8] US Department of Energy's National Renewable Energy Laboratory (NREL). Best Research-Cell Efficiency Chart. Retrieved from https://www.nrel.gov/pv/cell-efficiency.html. Accessed 9 February 2023.

[9] Wikipedia contributors. Solar-Cell Efficiency. Retrieved from https://en.wikipedia.org/wiki/Solar-cell_efficiency. Accessed 9 February 2023.

6 Companies

	Contact	Description	Price (inc. VAT)	Notes
JGJ Electrical	Johnny Koen. 074-090-3560, 072-910-7133. Bryanston. `jgjelectrical86@gmail.com`			Johnny and his company seem to be general electrical contractors, but also do solar installations. Johnny Koen was highly recommended on the I Love Facebook FB group.
Lumax Energy	010-140-4933. Unit 1, 270 Roan Crescent, Corporate Park North, Midrand. `energy@lumaxenergy.com`, `www.lumaxenergy.co.za`			They provide solar mounting solutions. Their website indicates that they, inter alia, build carports specifically to support solar panel arrays. Approved Solar uses them.
SunStore	010-449-2010. 067-412-1862 (WhatsApp). Warehouse A26 Co.Space, Cnr. Brand Road & Swart Drive, President Park AH, Midrand. `sales@sunstore.co.za`, `www.sunstore.co.za`	Solar Kit 4a "Sunsynk-Kilo" 4.2 kWp 48 V: 5 kW (6.25 kVA) 48 V SunSynk hybrid inverter. 1×5 kWh I-G3N $LiFePO_4$ battery. 9×450 W solar panels. **Installation not included.**	94.9k	SunStore is an online store supplying solar hardware. But they also do installations if needed. Use their website online store to ratify hardware pricing quoted by other installers. It is a division of Amperion Energy, a black women-owned business.
		Solar Kit 5a "Sunsynk-Mega" 5 kWp 48 V: As above, but $11 \times$ solar panels.	104.9k	

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	Contact	*Description*	*Price (inc. VAT)*	*Notes*
FM Solar	Nick @ 084-062-2971 (JHB), 061-524-3191 (PTA), 010-595-3114 (Benoni). 23 Skyway Business Park, Cnr CR Swart Drive & Freda Rd, Strydompark. `www.fmsolar.co.za`, `nick@fmsolar.co.za`	5 kW Deye hybrid inverter. 1 × 5.12 kWh 48 V Dyness lithium ion battery. 8 × 455 W JA solar panel. Installation, CoC.	135k	Their website's content is very well written. Website says will set up an initial meeting on potential client's premises. Website has a nice informative FAQ section. Nick is the JHB branch manager. From his personal FB page, he is *not* technical.
		8 kW Synsynk hybrid inverter. 1 × 5.32 kWh 51.2 V Synsynk LFP lithium ion battery. 12 × 455 W JA solar panel. Installation, CoC.	200k	
LCP Solar	Lyndsay Cotton @ 012-811-0452, 086-557-1913, 082-564-2730. Plot 303, Zwavelpoort, Graham Rd M6, Pretoria East. `lyndsay@lcproofing.co.za`, `quote@lcproofing.co.za`. `www.lcproofing.co.za/lcp-solar/`.			LCP Solar is a spin-off of LCP Roofing, so they might be able to advise on overcoming our thatch roof constraint. Website is nice in that it is very informative about PV and of their workflow from pre-quote till CoC. The website is nicely technical.

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	Contact	Description	Price (inc. VAT)	Notes
El Solar Solutions	Leon @ 078-228-3810. Elmarie @ 082-563-0326. 170 Soutpansberg Dr, Van Riebeeck Park, Kempton Park. `www.facebook.com/ELsolarS`	4 kW (5 kVA) Deye inverter. 1 × 5.1 kWh LBSA lithium ion battery. 6 × 455 W Canadian mono panel. Installation, CoC.	95.9k	No website. Saw advert on FB.
Emco Solar Solutions	Chris @ 081-824-9078. Tannery Industrial Centre, 308 Derdepoort Rd, Silverton, Pretoria. `www.emco-solar.co.za`, `info@emco-solar.co.za`	5 kW Sunsynk hybrid inverter. 1 × 5 kWh Raystech battery. 6 × Jinko 550 W panels. DC DB incl lightning arrestors. AC DB incl changeover switch. Installation, CoC, PV Greencard.	106k	Since 2015. Subsidiary of EMC Contracting. Saw on FB.
		5 kW Growcall hybrid inverter. 1 × 5.12 kWh lithium ion battery. 6 × 550 W mono panel. DC DB incl lightning arrestors. AC DB incl changeover switch. Installation, CoC, PV Greencard.	98.5k	
		8 kW Growcall hybrid inverter. 2 × 4.8 kWh lithium ion battery. 8 × 550 W mono panel. DC DB incl lightning arrestors. AC DB incl changeover switch. Installation, CoC, PV Greencard.	142.5k	
		As above, but Synsynk not Growcall.	152.5k	

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	Contact	Description	Price (inc. VAT)	Notes
Solar Man	012-809-1525, 071-690-8031. WhatsApp call: 071-555-3803 or 071-690-8031. Shop 2, Willow Rock Value Centre, c/o Solomon Mahlangu Drive & Bendeman Boulevard, Pretoria. `www.solarmansa.co.za`, `info@solarmansa.co.za`			The website suggests a visit to their store to talk with a consultant. Supplies Schubart $LiFePO_4$ batteries and Schubart 6-GFM-240J gel batteries.
Eagle Solar	Jannie Krugel @ 082-302-6773, 078-377-2366 (technical), René Krugel @ 071-135-2751 (marketing & admin), Carl Gersback @ 082-323-2099. HO: 79 Caledon Street, Standerton. Have a branch in Kempton Park. `jannie@eaglewirelesssecurity.co.za`, `info@eaglewirelesssecurity.co.za`. `www.eaglews.co.za`, `www.eaglewirelesssecurity.co.za`, `https://web.facebook.com/groups/507850296991519`.	5 kW LuxPower inverter. 1×5.1 kWh Green Rich battery. 6×560 W JA solar panel. Installation, CoC, PV Greencard. Seen on FB.	109.9k	Impressively written company profile PDF. Seems to be a spin-off off Eagle Wireless Security and Electrical. Product brands: Fivestar Solar, Canadian Solar, CBI Solar, JA Solar, Victron, Growatt, Axpert Allgrand, Goldshine Batteries, Lithium Ion, Pylon Tech, Blue Nova, Revo, Narada, Shoto, Sun Pays.

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	Contact	Description	Price (inc. VAT)	Notes
		5.5 kW Synsynk hybrid inverter. 1 × 5.4 kWh BSL li battery. 8 × 455 W Longi solar panel. Installation, CoC, PV Greencard. Seen on FB.	129.9k	
		5 kW Synsynk inverter. 2 × 5.1 kWh BSLA battery. 10 × 545 W JA solar panel. Installation, CoC, PV Greencard. Seen on FB.	159.9k	
		5 kW Deye inverter. 1 × 5 kWh Pylontech battery. 6 × 540 W Canadian solar panel. Installation, CoC, PV Greencard. Seen on website.	169.9k	
		8 kW Deye inverter. 1 × 7 kWh BSL Battery. 16 × 455 W Longi solar Panel. Installation, CoC, PV Greencard. Seen on website.	169.9k	

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	Contact	Description	Price (inc. VAT)	Notes
		8 kW Deye inverter. 1 × 8.8 kWh BSL battery. 12 × 545 W Canadian solar panel. Installation, CoC, PV Greencard. Seen on FB.	199.9k	
		8 kW Deye inverter. 2 × 5.1 kWh HiOn ($LiFePO_4$) battery. 12 × 545 W Canadian solar panel. Installation, CoC, PV Greencard. Seen on FB.	209.9k	
		8 kW Deye inverter. 2 × 5.4 kWh BSL lithium ion battery. 12 × 545 W Canadian solar panel. Installation, CoC, PV Greencard. Seen on FB.	219.9k	
		8 kW Deye inverter. 1 × 10 kWh Freedom Won lithium ion battery. 14 × 600 W Canadian solar panel. Installation, CoC, PV Greencard. Seen on website.	219.9k	

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	Contact	Description	Price (inc. VAT)	Notes
		8 kW SunSync inverter. 2 × 5.1 kWh SunSync battery. 14 × 600 W Canadian solar panel. Installation, CoC, PV Greencard. Seen on website.	219.9k	
Energy of Africa	010-335-0392, 064-455-3623, 061-739-1938. 14 Clair Road, Lynnwood Glen, Pretoria. `sales@energyofafrica.co.za`. `www.energyofafrica.co.za`	4 kW (5 kVA) hybrid inverter. 2 × 5.1 kWh lithium ion battery. 10 × 455 W panel.	160k	Website's three product offerings do not mention component brands names, but say they use "Tier 1" ones. FB page said can do an onsite energy audit and solar assessment.
		6.4 kW (8 kVA) hybrid inverter. 3 × 5.1 kWh lithium ion battery. 14 × 455 W panel.	240k	
		SRNE non-solar backup offering: 3.5 kW inverter. 1 × 5.1 kWh lithium ion battery.	35k	Can run for 4 hours: 20 × light, 2 × TV, 2 × laptop, 2 × fridge, 1 × kettle or microwave.

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	Contact	Description	Price (inc. VAT)	Notes
JHC Power Solutions	082-385-7231. 072-713-9088. Pretoria. `jhcpowersolutions@gmail.com`, `www.jhcpowersolutions.co.za`			Website includes nice detail about component brands and pricing, but no complete product offerings. But FB shows they're busy doing installations. And it seems that the owners themselves are doing the hands-on work.
Solar4You	012-345-1849. 076-025-9609 (WhatsApp). 590 Vacy Lyle Street, Elarduspark, Pretoria. `info@solar4you.co.za`, `www.solar4you.co.za`	4 kW (5 kVA) Magneto hybrid inverter. 1 × 5.1 kWh lithium ion battery. 4 × 555 W solar panel. Installation, CoC. From FB pamphlet on 21Nov22.	96.5k	Since 2008. Patented their Revolutionary Solar Geyser in 2017. Their website is stale but they post regularly on FB. A nice plus is that they have experience in solar geysers.
		As above, but 6.4 kW (8 kVA) Magneto hybrid inverter.	115.5k	
		4 kW (5 kVA) Magneto hybrid inverter. 2 × 5.1 kWh lithium ion battery. 6 × 555 W solar panel. Installation, CoC.	139k	
		As above, but 6.4 kW (8 kVA) Magneto hybrid inverter.	158k	

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	Contact	*Description*	*Price (inc. VAT)*	*Notes*
Full Cycle Facilities Management (FCFM)	Neil @ 072-868-1509. Glenvista, Johannesburg, South Africa. `neil@fullcyclefm.co.za`	8.8 kW Sunsynk Inverter. 4 × 5.32 kWh Sunsynk battery. 18 × JA panel.	300k	No website. Saw on FB. Operating only since Nov22.
Approved Solar	Barry @ 083-600-0307. 106 2nd Avenue, Melville. Unit 7, 15 5th St, Wynberg, Sandton. `barry@approvedsolar.co.za`, `www.approvedsolar.co.za`			Mom's retirement centre used them. The company was registered in 2019. Leon Besaans and Barry Maselle are the company directors. Leon Besaans is the technical person. He apparently has a Diploma in Mechanical Engineering Qualification from the then Pretoria Technikon. Barry Maselle is the sales person. He apparently has a degree in Business Science Finance from UCT. Their website does not show packages and pricing. Their FB presence suggests that they will do an obligation-free upfront onsite inspection.
Jason (Maurice's installer)	Jason @ 079-559-5058.			

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	Contact	*Description*	*Price (inc. VAT)*	*Notes*
NexSolar	087-135-8000. Epsom Downs Office Park, 13 Sloane Street, Bryanston, Johannesburg. `sales@nexsolar.co.za`, `www.nexsolar.co.za`			Their website is *very* minimal. No pricing, no packages, no component detail. Still, they claim on FB, over 200 installations in 2019.
Sable Energy	087-265-8744. 087-265-9584 (Centurion). Unit 23B, InsideOut Rockfields, 35 Drongo street, Rooihuiskraal, Centurion. 12 Power street, Isando, Kempton Park, 1600. Unit 30, The Foundry, Isando. `info@sable-energy.co.za`, `www.sable-energy.co.za`	8 kW (10 kVa) 48 V pure sine wave Axpert hybrid inverter. Built-in MPPT solar controller (80A). Built-in parallel card (up to 8 inverters). 1 × 10 kWh $LiFePO_4$ battery. Compatible with mains and generator power. 3-year warranty on unit.	82.95k	Their website is quite minimal. But I saw a pamphlet on FB. They seem to focus on a range of “modular” plug-n-play inverter backup systems more than on integrated solar systems.
Mizu Energy	Ruvern or Sirisha Mudaly @ 062-652-5321. 068-118-2214 (WhatsApp). `sales@mizuenergy.com`, `www.mizuenergy.com`	5 kW Luxpower inverter. 1 × 5 kWh Shoto battery. 6 × 540 W JA solar panel. Installation, CoC.	84k	Their website is very minimal, not very well constructed, and does not show any physical address. They have a very short FB timeline of about 1 week. FB claims Ruvern studied industrial engineering at UJ and works at Zest WEG Group in Johannesburg.

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	Contact	Description	Price (inc. VAT)	Notes
		2 × 5 kW Luxpower inverter. 2 × 5 kWh Shoto battery. 16 × 540 W JA solar panel. Installation, CoC.	145k	
NeoSwitch	011-040-5600. 43 4th Street, Wynberg, Sandton. `www.neoswitch.co.za`, `upgrade@neoswitch.co.za`	10 kW inverter. 2 × 5 kWh lithium ion battery. 8 × 540 W panel.	126.5k	Seems more about Internet and voice connectivity than about solar.
Blue Mountain Energy	011-085-2600. 245 Masjien Street, Strydompark, Randburg. `www.bluemountainpv.co.za`, `www.kodak.com/en`			Supplies Kodak solar products in SA. Website does include full kits and pricing. Targetting installers
SegenSolar	`www.segensolar.co.za`, `www.kodak.com/en`			Supplies Kodak solar products in SA. Website does include full kits and pricing. Targetting installers
Sunflex Energy				
SK Solar				
Solar-Sun				
Solarsynk				

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	Contact	Description	Price (inc. UAT)	Notes
Solar Finder				
Sensible Solar				
Creative Group Projects				
Affordable Power Solutions				
Hudaco Energy				

End.