Mathematics Is a Beautiful Thing
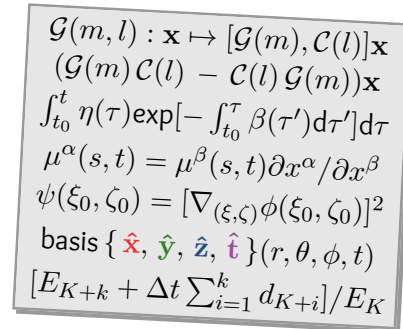
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$$\mathcal{G}(m,l) : \mathbf{x} \mapsto [\mathcal{G}(m), \mathcal{C}(l)]\mathbf{x}$$
$$(\mathcal{G}(m)\,\mathcal{C}(l) - \mathcal{C}(l)\,\mathcal{G}(m))\mathbf{x}$$
$$\int_{t_0}^{t} \eta(\tau)\exp[-\int_{t_0}^{\tau} \beta(\tau')\mathrm{d}\tau']\mathrm{d}\tau$$
$$\mu^{\alpha}(s,t) = \mu^{\beta}(s,t)\partial x^{\alpha}/\partial x^{\beta}$$
$$\psi(\xi_0, \zeta_0) = [\nabla_{(\xi,\zeta)}\phi(\xi_0, \zeta_0)]^2$$
$$\text{basis } \{\,\hat{\mathbf{x}},\ \hat{\mathbf{y}},\ \hat{\mathbf{z}},\ \hat{\mathbf{t}}\,\}(r, \theta, \phi, t)$$
$$[E_{K+k} + \Delta t \textstyle\sum_{i=1}^{k} d_{K+i}]/E_K$$

MATHEMATICS IS BEAUTIFUL. But whence comes its beauty? From its *utility* and its *surprise*.

Utility. Mathematics offers us to dance a delightful caper right on the boundary of what is known and unknown. It offers a conceptual bridge between the worldly mundane and the other-wordly exceptional, thereby keeping both Aristotle and Plato happy. Mathematics takes prosaic commonsense ideas and turns them into poetry. It transforms cloddish (ac)counting systems into intricate and emergent tapestries of meaning. With mathematics, we may begin to imagine the unimaginable, whilst remaining rooted in the real. And so, the vector space in which we are all immersed and go about our day-to-day days, becomes a particular case of something much richer.

Surprise. It is surprising, I think, that there are strictly as many rational numbers as there are integers, even though the set of integers forms a subset of the set of rationals. Wait, what? It is equally surprising that there are infinitely more irrational numbers than rational ones. By the way, irrational numbers, such as π, e and $\sqrt{2}$, are those which cannot be expressed as the ratio of two integers, and so whose decimal expansion never terminates.

Who would have thought that by obsessing over infinities and infinitesimals, calculus would come to tell stories of the evolution of finite instants? Oh the irony!

And who would have thought that the so-called imaginary numbers, such as the square root of -7, denoted $\sqrt{-7}$, and whose very existence seems impossible, would take centerstage in describing the strangeness of the real world—the quantum world in which, surprisingly, we all live?

Utility and surprise. But for me, the beauty of mathematics is exemplified in one of its most useful and surprising equations of all. Euler's Identity:

$$e^{\mathrm{i}\pi} + 1 = 0 \tag{1}$$

For in a single equation, five of Nature's most fundamental numbers are bound, as if by magic.

e is the base of natural logarithms. It embodies pure non-periodicity. It is irrational.

π embodies pure periodicity. It too is irrational.

i is the square root of -1, denoted $\sqrt{-1}$, and so is imaginary.

0 is the identity element of the so-called additive group over the integers.

1 is the identity element of the multiplicative group over the integers.

Utility and surprise! If there is a god (or gods), then mathematics would surely be its language. Not Hebrew, not Arabic, not Sanskrit. And not Tongues!