Is Mathematics Invented or Discovered?

Paul Kotschy

2 September 2022

Compiled on February 19, 2025

$$\mathcal{G}(m,l) : \mathbf{x} \mapsto [\mathcal{G}(m), \mathcal{C}(l)]\mathbf{x}$$
$$(\mathcal{G}(m)\,\mathcal{C}(l) - \mathcal{C}(l)\,\mathcal{G}(m))\mathbf{x}$$
$$\int_{t_0}^{t} \eta(\tau)\mathsf{exp}[-\int_{t_0}^{\tau} \beta(\tau')\mathsf{d}\tau']\mathsf{d}\tau$$
$$\mu^{\alpha}(s,t) = \mu^{\beta}(s,t)\partial x^{\alpha}/\partial x^{\beta}$$
$$\psi(\xi_0, \zeta_0) = [\nabla_{(\xi,\zeta)}\phi(\xi_0,\zeta_0)]^2$$
$$\mathsf{basis}\,\{\,\hat{\mathbf{x}},\,\hat{\mathbf{y}},\,\hat{\mathbf{z}},\,\hat{\mathbf{t}}\,\}(r,\theta,\phi,t)$$
$$[E_{K+k} + \Delta t \sum_{i=1}^{k} d_{K+i}]/E_K$$

Is mathematics invented or discovered? This question arises because there seem to be two sides to mathematics. One side is practical, and the other is conceptual.

In one sense, mathematics is a language. It has nouns, verbs and grammar, such as numbers, sets, operators and mappings. We as humans attach symbols to these entities, and we attribute meaning to these symbols in order to encode layers of emergent meaning, just as we do with natural languages and with computer languages. This is indeed an act of invention.

But in another sense, mathematics is more than a language. There seem to be objects in reality and relations between objects which exist completely independently of us and of our symbols. Our symbols are merely a way of elucidating what is already there. John Lennox refers to the apparent "rational intelligibility" of the Universe. Encoding this intelligibility is an act of discovery.

For example, to an engineer, quaternion numbers are useful for encoding three-dimensional rotations. To an engineer, quaternions were invented. But to a mathematician, quaternions form a so-called finite-dimensional associative division algebra over the real numbers. There are only three such division algebras, one of which is the set of quaternions. I think this is conceptually interesting. And it is true with or without us being around to contemplate it. To a mathematician, this is a discovery.

Another example is Euler's Equation:

$$e^{\mathrm{i}x} = \cos(x) + \mathrm{i}\sin(x)$$

In one sense, this is an equation of great utility. It is a useful invention, and so is widely used. But in another sense, the equation encapsulates something profound, namely, a deep connection between two opposing concepts—concepts which exist independently of us. These are the concepts of periodicity and non-periodicity. And the complex number 'i' in Euler's equation is the number which establishes that connection.