Fix That Leaking Tap!

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As a hot water tap drips, water which was previously heated in a geyser leaves the geyser and is lost down the drain. But what is also lost is the heat energy held by that hot water. As the hot water drips away, leaving the geyser, new water will have to be continually added to the geyser and heated, thereby requiring an input of extra energy, usually electrical energy.

I would like to determine the cost of this extra electrical energy. To do so, I must link the leaking water to a loss of heat energy, and that loss of heat energy to an electricity consumption, and that electricity consumption to a monetary cost.

Leak and heat. It is easy to measure the rate at which water drips from a tap. Simply place a container under the tap for a given length of time and measure the accumulated volume of water in the container. Let f_{v} be that number, namely, the volumetric flow rate of water.

Since the mass density of water, ρ, is well known, we may convert the volumetric flow rate to a water mass flow rate, f_{m}, as

$$f_{\mathrm{m}} = \rho f_{\mathrm{v}}$$

If we start our timer at the moment we place the container under the leaking tap, we know that at any subsequent time t, the mass of water that would have left the geyser must be

$$m(t) = f_{\mathrm{m}} t = \rho f_{\mathrm{v}} t \tag{1}$$

A geyser raises the temperature of a body of water by adding heat energy to the water. For a body of water of mass $m(t)$, that amount of heat energy added must be

$$Q(t) = m(t) c \left(T_{\mathrm{h}} - T_{\mathrm{amb}}\right) \tag{2}$$

where c is the specific heat capacity of water in the temperature range $[T_{\mathrm{amb}}, T_{\mathrm{h}}]$, T_{h} is the configured hot water temperature of the geyser, and T_{amb} is the ambient temperature of the water entering the geyser. Equation (2) also gives the amount of heat energy lost at time t as a result of the tap dripping throughout the time interval $[0, t]$. Therefore, combining (1) and (2) gives the heat lost as

$$Q_{\mathrm{drip}}(t) = \rho c f_{\mathrm{v}} (T_{\mathrm{h}} - T_{\mathrm{amb}}) t \tag{3}$$

Consume electricity. How much electricity was used to produce an amount $Q_{\text{drip}}(t)$ of heat energy? The answer is an amount of electricity required to produce *any* energy equal to $Q_{\text{drip}}(t)$. The obvious reason for this is that electricity consumption does not discriminate over the form of energy used. So if W is the overall electrical energy used in a household per unit time, and if B is the cost of electricity for a household per unit time, then B/W must be the cost of electricity per unit of electrical energy consumed in the household. Therefore, using (3), the cost at time t of a dripping hot water tap is

$$\boxed{h(t) = \frac{B}{W} Q_{\text{drip}}(t) = \frac{B\rho c f_{\text{v}}(T_{\text{h}} - T_{\text{amb}})}{W} \cdot t} \tag{4}$$

Gather data. The following water flows were measured at the dripping tap.

Hours	*Volume Dripped* [l]	*Volumetric Flow Rate* [l/h]
1.5	1.325	0.883
1.08333	0.91	0.84
0.7333	0.62	0.845
0.9666	0.8	0.827
1.1333	0.91	0.802
1.3	1.0	0.769
0.6333	0.485	0.765
1.6	1.19	0.743
1.9166	1.415	0.738
10.866	8.655	*Average*: 0.796

So the average volumetric drip flow rate is

$$f_{\text{v}} = 0.796 \text{ l/h} = 0.796 \times 10^{-3} \text{ m}^3\text{/h} = 2.212 \times 10^{-7} \text{ m}^3\text{/s} \tag{5}$$

Between 20Jan23 and 16Feb23, we recorded our electrical energy usage as follows:

Date	*Time*	*Hours*	*Meter Reading* [kWh]	*Energy Use* [kWh]
20Jan23	03h40		620.51	
21Jan23	04h50	25.167	610.19	10.32
23Jan23	04h30	47.667	579.36	30.83
24Jan23	09h45	29.25	563.80	15.56
25Jan23	06h45	21.0	553.39	10.41
26Jan23	03h50	21.083	539.14	14.25
27Jan23	04h30	24.667	520.35	18.79
3Feb23	09h45	173.25	409.22	111.13
9Feb23	22h30	156.75	304.03	105.19
16Feb23	20h08	165.633	202.86	101.17
		Total: 664.467		*Total*: 417.65

The average electrical energy consumed per unit time is therefore

$$W = \frac{417.65 \text{ kWh}}{664.467 \text{ h}} = 628.548 \text{ W} \tag{6}$$

Over a five month period our household electricity costs were:

Date	*Cost* [R]
11Mar23	
11Apr23	2000
15May23	2000
13Jun23	2000
5Jul23	2000
27Jul23	2000
12Aug23	2000

From these data, the average cost of our household electricity per unit time is

$$B = \frac{6 \times \text{R}2000}{22\ \text{w}} = 545.454\ \text{R/w} = 3.246\ \text{R/h} = 9.018 \times 10^{-4}\ \text{R/s} \tag{7}$$

And so the average cost per unit of energy consumed must be

$$\frac{B}{W} = \frac{9.018 \times 10^{-4}\ \text{R/s}}{628.548\ \text{W}} = 1.434 \times 10^{-6}\ \text{R/J}$$

The ambient temperature of cold water entering the geyser and the hot temperature of the water leaving the geyser are

$$\begin{aligned} T_{\text{amb}} &= 15\,^{\circ}\text{C} \\ T_{\text{h}} &= 60\,^{\circ}\text{C} \end{aligned} \tag{8}$$

In the temperature range $[15\,^{\circ}\text{C}, 60\,^{\circ}\text{C}]$ the mass density of water is about

$$\rho = 990\ \text{kg/m}^3 \tag{9}$$

And in the same temperature range, the specific heat capacity of water is about

$$c = 4184\ \text{J/kg/}^{\circ}\text{C} \tag{10}$$

The cost. Substituting (5)…(10) into (4) we are able to calculate the cost of the dripping hot water tap as

$$\boxed{\begin{aligned} h(t) &= 5.917 \times 10^{-5} \cdot t && \text{in units R, s} \\ &= 0.213 \cdot t_{\text{h}} && \text{in units R, h} \\ &= 5.112 \cdot t_{\text{d}} && \text{in units R, d} \\ &= 155.507 \cdot t_{\text{m}} && \text{in units R, mo} \end{aligned}}$$