Monetary Net Worth and the Relationships between Savings, Income, Expenses, Tax, Borrowings, and the Sale of Asset such as a Business

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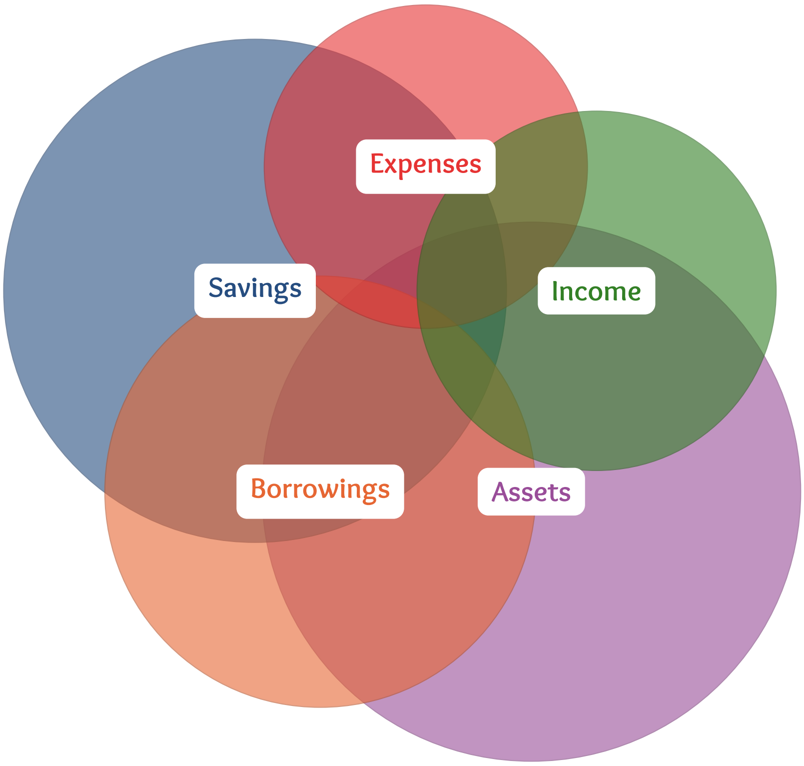

Abstract

Some of the variables which play a role in determining an investor's financial circumstance are: income from labour, living expenses, investment savings, tax, borrowings, and the possession of assets. In this article, the role contribution of each of these variables to an investor's net monetary worth is analysed. The analysis is applied to two specific possible events in an investor's experience, namely, the event of retirement, and the sale of a business. By coupling these two events, the analysis will help to answer two questions: 1. *What minimum net monetary worth will enable an investor to retire?* And 2. *How do the abovementioned variables govern a buy-sell transaction of a business in which both the seller's and the buyer's interests are accounted for?*

Contents

1 Introduction

HOW ARE WE DOING FINANCIALLY? Are we able to retire? If not now, when? If we sell an asset such as our business, how will it alter our overall financial circumstance? How would we even value our business in the first place? And if we do sell it, will we then be able to retire? For many, these are important financial questions. In this article, I offer some of my own answers.

Some of the variables playing important roles in determining financial circumstance are: income from labour, living expenses, investment savings, tax, borrowings, and the possession of assets. These variables each play a role in setting a *monetary balance*, wherein an investor's net monetary worth at a given time is the sum of savings, income and assets less expenses.

Given a monetary balance at a certain time, we wish to be able to predict the monetary balance at future times. Furthermore, we may wish to simulate certain singular events in an investor's financial trajectory. Such events could include: the event of retirement, the purchase of an asset, the sale of a business, or the purchase of a business together with a set of payments for the business over time. By carefully analysing the monetary balance, and by imposing one or more constraints on it, we are able to simulate these events. These simulations help us to make better decisions for the future.

2 Net Monetary Worth

WE BEGIN WITH THE *monetary balance equation*. Let the net *income* from labour be denoted by I, investment *savings* be denoted by S, value of *assets* by A, and living *expenses* by E. Since net income from labour is usually incremented month by month in the form of a net salary, it is reasonable to use the month as the basic unit of time. Corresponding quantities are then denoted I_n, S_n, A_n and E_n for month n. An investor's *net monetary worth* at month n, denoted by W_n, is:

$$\boxed{W_n = S_n + A_n + I_n - E_n} \tag{1}$$

This net monetary worth, W_n, is an all important quantity in assessing financial circumstance for the investor. How does W_n vary over time? How do specific singular events impact W_n, such as the event of retirement? Importantly, how do changes in S_n, A_n, I_n and E_n over time contribute to changes in W_n? And what about inflation, that subliminal and apparently inevitable erosion of the value of W_n over time?

For reasons to do with my own immediate personal circumstance, I ignore for now any tangible assets in the ensuing analysis. I therefore choose to set $A_n = 0$ for all n.

If enough is known at the end of some month to be able to compute a net monetary worth, then we may as well start the quantitative analysis at the end of that month and label it 'month 0'. That is, we know what S_0, I_0 and E_0 are. If we also know their respective rates of change from month 0 to month 1, then we may write:

$$\begin{aligned} E_1 &= (1+\alpha)E_0 \equiv eE_0 \\ S_1 &= (1+\beta)S_0 \equiv sS_0 \\ I_1 &= (1+\gamma)I_0 \equiv iI_0 \end{aligned} \tag{2}$$

where the quantities α, β and γ are, respectively, the *inflation rate*, the investment *savings rate*, and the *income growth rate* per month, i.e., $\frac{1}{12}$ the annual rates.

It is now assumed that the investor is a prudent saver. A prudent saver will attempt to maximise net worth, W_n, by injecting the difference $I_n - E_n$ at the end of month n into the savings. In this way, under the assumption that $A_n = 0$, the previous month's savings and net worth must be equated:

$$S_{n-1} := W_{n-1} \tag{3}$$

Equations (1), (2) and (3) give for month 1

$$\begin{aligned} W_1 &= S_1 + I_1 - E_1 \\ &= sS_0 + iI_0 - eE_0 \\ &= sW_0 + iI_0 - eE_0 \end{aligned}$$

Similarly, for month 2:

$$\begin{aligned} W_2 &= sW_1 + iI_1 - eE_1 \\ &= s(sW_0 + iI_0 - eE_0) + i^2 I_0 - e^2 E_0 \\ &= s^2 W_0 + i(s+i)I_0 - e(s+e)E_0 \end{aligned}$$

For month 3:

$$\begin{aligned} W_3 &= sW_2 + iI_2 - eE_2 \\ &= s(s^2 W_0 + i(s+i)I_0 - e(s+e)E_0 + i^3 I_0 - e^3 E_0 \\ &= s^3 W_0 + i(s^2 + is + i^2)I_0 - e(s^2 + es + e^2)E_0 \end{aligned}$$

Therefore, for month n:

$$\begin{aligned} W_n &= sW_{n-1} + iI_{n-1} - eE_{n-1} \\ &= s^n W_0 + i^n \sum_{j=1}^{n} \left(\frac{s}{i}\right)^{j-1} I_0 - e^n \sum_{j=1}^{n} \left(\frac{s}{e}\right)^{j-1} E_0 \end{aligned} \tag{4}$$

The series in Eq. (4) are geometric series for which the solution is:

$$\boxed{W_n = s^n W_0 + (s^n - i^n)\frac{i}{s-i} I_0 - (s^n - e^n)\frac{e}{s-e} E_0} \tag{5}$$

3 The Event of Retirement

Retirement may be defined as the condition in which the investor no longer earns income from labour, i.e., the condition $I_n = 0$ for all $n > n_\mathrm{R}$ for some month n_R of retirement. The decision as to when to retire should thus be governed by the imposition of one or more constraints on W_n.

One such constraint on W_n could be that the investor's net monetary worth vanishes after some month $n_\mathrm{D} > n_\mathrm{R}$. That is, the constraint is

$$W_{n_\mathrm{D}} = 0 \qquad \text{for some } n_\mathrm{D} > n_\mathrm{R}$$

Of course, the month n_D must be carefully selected by the investor to ensure that the investor is prepared to accept the vanishing of W_{n_D}.

An alternative constraint on W_n could be that subsequent to the month of retirement, n_R, the investor's net monetary worth grows at a rate greater than or equal to the rate of inflation. That is:

$$\left.\begin{aligned} W_{n+1} &\geq (1+\alpha)W_n = eW_n \\ I_n &= 0 \end{aligned}\right\} \text{ for all } n > n_\mathrm{R} \tag{6}$$

Using Eq. (1),

$$\begin{aligned} W_{n+1} &= S_{n+1} + I_{n+1} - E_{n+1} \\ &= S_{n+1} - E_{n+1} \end{aligned}$$

so that constraint Eq. (6) becomes

$$
\begin{aligned}
& S_{n+1} - E_{n+1} \geq eW_n \\
\Rightarrow \quad & sS_n - eE_n \geq eW_n \\
\Rightarrow \quad & \boxed{\begin{array}{ll} W_n \geq \dfrac{e}{s-e} E_n = \dfrac{e^{n+1}}{s-e} E_0 & \text{for all } n > n_{\mathrm{R}}, s > e \\ I_n = 0 & \end{array}}
\end{aligned}
\tag{7}
$$

Equation (7) expresses the intuitive idea that net worth will always grow at a rate faster than inflation, provided that expenses are curtailed and that the savings rate, $\beta = s - 1$ (Eq. (2)), is maximised.

But if the investor's current net monetary worth does not satisfy constraint Eq. (7), then we wish to know when in the future it will be satisfied, if at all. We seek to relate the current worth, say, W_0, to that of a future month n_{R} using the future-dated retirement criterion:

$$
W_{n_{\mathrm{R}}} \geq \frac{e}{s-e} E_{n_{\mathrm{R}}} \tag{8}
$$

Applying Eq. (5) at month n_{R} gives

$$
s^{n_{\mathrm{R}}} W_0 + (s^{n_{\mathrm{R}}} - i^{n_{\mathrm{R}}}) \frac{i}{s-i} I_0 - (s^{n_{\mathrm{R}}} - e^{n_{\mathrm{R}}}) \frac{e}{s-e} E_0 \geq e^{n_{\mathrm{R}}} \frac{e}{s-e} E_0
$$

so that

$$
W_0 \geq \frac{e}{s-e} E_0 - \frac{s^{n_{\mathrm{R}}} - i^{n_{\mathrm{R}}}}{s^{n_{\mathrm{R}}}} \frac{i}{s-i} I_0
$$

or

$$
W_0 \geq \frac{e}{s-e} E_0 - (1 - (i/s)^{n_{\mathrm{R}}}) \frac{i/s}{1 - i/s} I_0 \tag{9}
$$

Equation (9) prescribes a lower bound for an initial net monetary worth satisfying criterion Eq. (8) at some future month n_{R}. It is not surprising that W_0 is a decreasing function of that month n_{R}, the investor's current net income I_0, and the income's monthly growth rate $\gamma = i - 1$.

Conversely, instead of prescribing a lower bound for W_0, we may use (9) to prescribe a lower bound for the future month of retirement n_{R} as a function of W_0:

$$
n_{\mathrm{R}} \geq \frac{\log \left[1 + \dfrac{1 - i/s}{i/s} \dfrac{W_0}{I_0} - \dfrac{e}{i} \dfrac{1 - i/s}{1 - e/s} \dfrac{E_0}{I_0} \right]}{\log \left[i/s \right]} \tag{10}
$$

4 Selling a Business

THE OWNER OF A BUSINESS may wish to sell the business in order to augment the owner's net monetary worth, W_0, possibly with a view to retiring. Potential buyers of the business may prefer one of two options: 1. that the present owner leave the business completely; or 2. that the present owner remains in the employ of the business for some time after the sale.

In the case of the first option, because the owner retires at the current month ($n = 0$), the selling price for the business must be determined in part by the retirement criterion at month 0. That is Eq. (7) with $n = 0$ must be applied. But in the case of the second option, the present owner would expect to obtain income for labour for some period $[0, n_{\mathrm{R}}]$ after the sale at month 0. The selling price for the business must then be determined in part by the retirement criterion applied at the future month n_{R}. That is Eq. (9) must be applied.

So much for the seller of the business. What about the buyer? A potential buyer's willingness to purchase a business may be influenced by the buyer's burden in servicing a loan from a financial intermediary. Indeed, since the buyer would rely on future business earnings to service such a loan, there is a connection between the size of the loan and a buy-sell transaction value. We therefore turn now to an analysis of servicing such a loan for the purpose of purchasing a business.

5 Obtaining a Loan

A FINANCIAL INTERMEDIARY lends money on condition that the money is refunded with interest, obviously. The well-known compound interest equation governs this transaction. A time-discrete version of it is derived as follows. For any month n subsequent to the commencement of the loan, we must have

$$L_{n+1} = L_n + \rho L_n - P = (1+\rho)L_n - P \equiv lL_n - P$$

where L_n is the value of the loan at the end of month n; ρ is the lending rate, probably linked to the prime lending rate; $l \equiv 1+\rho$; and P is the monthly loan repayment amount which is assumed to be fixed over the loan period. If the initial value of the loan is L_0, then

$$\begin{aligned}
L_1 &= lL_0 - P \\
L_2 &= lL_1 - P = l^2 L_0 - \sum_{j=1}^{2} l^{j-1} P \\
&\vdots \\
L_n &= l^n L_0 - \sum_{j=1}^{n} l^{j-1} P
\end{aligned}$$

The solution is

$$L_n = l^n L_0 - \frac{1-l^n}{1-l} P$$

Therefore, if the financial intermediary requires the loan to be repaid after n_{L} months, say, then $L_{n_{\mathrm{L}}} = 0$, and the value of L_0 is constrained by

$$L_0 = \frac{1-l^{n_{\mathrm{L}}}}{l^{n_{\mathrm{L}}}} \frac{1}{1-l} P$$

Conversely, an initial loan valed at L_0 will be fully repaid after n_{L} months provided that the monthly loan repayment satisfies

$$\boxed{P = \frac{l^{n_{\mathrm{L}}}(1-l)}{1-l^{n_{\mathrm{L}}}} L_0} \tag{11}$$

As expected, the monthly loan repayments increase with increase in the initial borrowed amount, L_0.

The seller of the business (Section 4) may request all or part of the payment for the business to be made as a lump sum payment. The buyer may need to borrow funds to meet this request. The next section elaborates on this by coupling Eqs. (5) and (7) or (8) to (11).

6 Lump Sum Payment followed by monthly Payments

IN THIS SECTION, I investigate the feasability of the present owner of a business selling the business in order to receive an initial lump sum payment, L_0, followed by a remaining payment spread over n_{R} months in equal monthly payments, Q. The lump sum payment, L_0, would augment, in part, any initial net monetary worth, K_0, of the seller.

6.1 Tax

We would have hoped that all of L_0 contributes to the seller's W_0. Unfortunately, the levying of various types of taxes during the transaction serves to complicate what is superficially a simple buy-sell scenario. These taxes are not neglible, and must therefore be accounted for accurately. The different taxes are:

- Value-added tax (VAT).
- Standard income tax on individuals, levied as a fraction of the individuals' gross taxable income.
- Company income tax, levied as a fraction of a company's gross profit after VAT has been paid.
- Secondary tax on companies (STC), levied not as a fraction of a company's net profit which is available for distribution as a dividend to shareholders, but instead, as a fraction of the dividend itself which has just been distributed.
- Capital gains tax (CGT) on individuals, levied as a fraction of the value of an asset owned by an individual and which has just been sold.
- Capital gains tax on companies, levied as a fraction of the value of an asset owned by a company and which has just been sold.

Consideration of these taxes forces us to consider two broad buy-sell scenarios. In the first scenario, the seller's shareholding in the seller's business (i.e., the seller's interest in the business) is sold to one or more individual buyers. In the second scenario, the seller's business assets and liabilities are sold to a new business. The seller's business remains as is but ceases to trade because all trading is done in the new business. We analyse both scenarios here.

6.2 Selling the Member's Interest in a Business

In the first scenario, the buyer and seller agree that the buyer purchase the seller's interest in the business, rather than the business itself. That is, the assets and liabilities remain in the business, and the buyer individually honours the payments P to the lender and Q to the individual seller. However, the buyer would likely need his or her salary to be raised in order to make the monthly payments. Unfortunately, raising the buyer's salary introduces additional individual income tax, especially if the buyer's gross salary shifts into a higher income tax range.

On its own, this required increase in buyer's gross salary imposes a stiff penalty on the transaction. However, the increase also serves to reduce the business's gross profitability, which in turn, reduces the 'Secondary Tax on Companies' tax—a tax levied on any net company profits which are distributed as a dividend.

In this scenario, Capital Gains Tax (CGT) is imposed on the lump sum payment before the seller can add it to W_0. That is

$$W_0 = K_0 + (1 - \tau_{\text{icgt}})L_0 \equiv K_0 + t_{\text{icgt}}L_0$$

where τ_{icgt} is the CGT rate applied to individuals, and $t_{\text{icgt}} \equiv 1 - \tau_{\text{icgt}}$ is defined for convenience.

The lump sum payment to the seller would presumably be serviced by the buyer via a loan, in the manner described in Section 5. It is assumed that the monthly loan repayments, P, and the monthly payments to the seller, $t_{\text{icgt}}Q$, are made concurrently, spread over the same term spanning n_{R} months.

If we also assume that the seller's monthly income from labour, I_n, during the term is nil, then it is easy to show, taking the loan L_0 and the n_{R} payments of Q into account, that the expression in Eq. (5) for the seller's net monetary worth at month n_{R} becomes

$$W_{n_{\text{R}}} = s^{n_{\text{R}}}(K_0 + t_{\text{icgt}}L_0) + \frac{s^{n_{\text{R}}} - 1}{s - 1} t_{\text{icgt}}Q - e\frac{s^{n_{\text{R}}} - e^{n_{\text{R}}}}{s - e}E_0 \tag{12}$$

Imposing the retirement criterion, Eq. (7), at month n_{R} gives a criterion for the payments, Q:

$$\boxed{Q \geq \frac{1}{t_{\text{icgt}}} \frac{s^{n_{\text{R}}}(s-1)}{s^{n_{\text{R}}} - 1} \left[\frac{e}{s-e}E_0 - K_0 - t_{\text{icgt}}L_0 \right]} \tag{13}$$

This expresses the wishes of the seller in terms of the seller's context (E_0 and K_0), the transaction (L_0), and taxes (t_{icgt}). Obviously, the total payment received by the seller is

$$Q_{\text{tot}} = t_{\text{icgt}}(L_0 + n_{\text{R}}Q)$$

If the buyer's *gross* salary is currently Y_G, and the buyer's individual tax rate is currently τ_i, say, then the buyer's current *net* salary relates to gross salary by

$$Y_\mathsf{G} = Y_\mathsf{N} + \tau_\mathsf{i} Y_\mathsf{G} \tag{14}$$

Similarly, if the buyer's salary were to increase to $Y_\mathsf{G}{}'$, it would be taxed at a new individual tax rate $\tau_\mathsf{i}{}'$, so

$$Y_\mathsf{G}{}' = Y_\mathsf{N}{}' + \tau_\mathsf{i}{}' Y_\mathsf{G}{}' \tag{15}$$

But since the sole purpose of increasing the buyers's salary is to assist the buyer in honouring the monthly payments, P, to the financial intermediary, and to the seller, Q, the buyer's new net salary must relate to the buyer's current net salary by

$$Y_\mathsf{N}{}' = Y_\mathsf{N} + P + Q \tag{16}$$

Substituting Eqs. (14) and (15) into Eq. (16), together with the convenience definitions $t_\mathsf{i} = 1 - \tau_\mathsf{i}$ and $t_\mathsf{i}{}' = 1 - \tau_\mathsf{i}{}'$, it is easy to show that

$$\boxed{Y_\mathsf{G}{}' = \frac{t_\mathsf{i}}{t_\mathsf{i}{}'} Y_\mathsf{G} + \frac{1}{t_\mathsf{i}{}'}(P + Q)} \tag{17}$$

The business's current *gross* monthly profit before and after the buy-sell transaction, and after the usual Value Added Tax (VAT) has been deducted, may be expressed as[1].

$$\begin{aligned} R_\mathsf{G} &= I_\mathsf{G} - E_\mathsf{G} = I_\mathsf{G} - (\widetilde{E}_\mathsf{G} + Y_\mathsf{G}) \\ R_\mathsf{G}{}' &= I_\mathsf{G}{}' - (\widetilde{E}'_\mathsf{G} + Y_\mathsf{G}{}') \end{aligned} \tag{18}$$

where I_G is the business's gross monthly income, and $\widetilde{E}_\mathsf{G}$ is the sum of all current monthly expenses less the buyer's current gross salary. Company tax is imposed on these profits at a rate τ_c, so that

$$R_\mathsf{N}{}' = R_\mathsf{G}{}' - \tau_\mathsf{c}{}' R_\mathsf{G}{}' \equiv t_\mathsf{c}{}' R_\mathsf{G}{}' = t_\mathsf{c} R_\mathsf{G}{}' \tag{19}$$

This *net* company profit is often termed the *dividend cover* because it is out of this amount that dividends may be distributed.

While company tax is imposed on gross company profits (Eq. (19)), the Secondary Tax on Companies (STC) tax is imposed, at a rate τ_stc, on the actual dividends which are distributed. And of course, it is the distributed dividends which are of interest, because it is only through them that the new buyer is able to affect his or her net monetary worth during the payment term. Recall from Eq. (16) that the buyer's effective net salary does not change.

Assuming that under the transaction all of $R_\mathsf{N}{}'$ is distributed as a dividend, D', then

$$D' = R_\mathsf{N}{}' - \tau_\mathsf{stc} D'$$

so that

$$R_\mathsf{N}{}' = (1 + \tau_\mathsf{stc}) D' \equiv t_\mathsf{stc} D' \tag{20}$$

Eqs. (19) and (20) permit the definition of an *Effective Company Tax* imposed on gross company profits leading to a distributed dividend as

$$\begin{aligned} D &= R_\mathsf{G} - \tau_\mathsf{ec} R_\mathsf{G} \equiv t_\mathsf{ec} R_\mathsf{G} \\ D' &= R_\mathsf{G}{}' - \tau_\mathsf{ec} R_\mathsf{G}{}' \equiv t_\mathsf{ec} R_\mathsf{G}{}' \end{aligned} \tag{21}$$

where

$$\tau_\mathsf{ec} \equiv \frac{\tau_\mathsf{c} + \tau_\mathsf{stc}}{1 + \tau_\mathsf{stc}} \quad \text{and} \quad t_\mathsf{ec} \equiv 1 - \tau_\mathsf{ec}$$

We now seek to relate the distributed dividend, D', to the details of the transaction, i.e., to P, Q, and to the business's current gross profitability. If after the transaction, the seller's gross salary is reduced to zero, it effectively becomes available to help reduce the business's monthly expenses. That is (c.f. Eq. (18)),

$$\widetilde{E}'_\mathsf{G} = \widetilde{E}_\mathsf{G} - Y_\mathsf{Gseller}$$

[1] I use the letter 'R' for profit because 'P' is already used and the notion 'profit' reminds me of 'residual'

We may also conservatively assume that after the transaction the business's sources of income remain unchanged, i.e., $I_{\mathsf{G}}' = I_{\mathsf{G}}$. From Eq. (18) we therefore obtain

$$R_{\mathsf{G}}' = R_{\mathsf{G}} + Y_{\mathsf{Gseller}} - \frac{t_{\mathsf{i}} - t_{\mathsf{i}}'}{t_{\mathsf{i}}'} Y_{\mathsf{G}} - \frac{1}{t_{\mathsf{i}}'}(P + Q)$$

Using Eq. (11), the business's new net profitability may be expressed in terms of the business's existing context (R_{G}, Y_{Gseller} and Y_{G}), the transaction (L_0 and Q) and taxes (t_{i}, t_{i}' and t_{c}), as

$$\boxed{R_{\mathsf{N}}' = t_{\mathsf{c}} \left[R_{\mathsf{G}} + Y_{\mathsf{Gseller}} - \frac{t_{\mathsf{i}} - t_{\mathsf{i}}'}{t_{\mathsf{i}}'} Y_{\mathsf{G}} - \frac{1}{t_{\mathsf{i}}'} Q - \frac{1}{t_{\mathsf{i}}'} \frac{l^{n_{\mathsf{R}}}(1 - l)}{1 - l^{n_{\mathsf{R}}}} L_0 \right]} \tag{22}$$

In the interests of the business, we stipulate that the business must not operate at a loss as a result of the buy-sell transaction. That is:

$$\boxed{R_{\mathsf{N}}' \geq 0}$$

so that $R_{\mathsf{G}}' \geq 0$, with which Eq. (22) may be used to place an upper bound on the monthly payments to the seller over the payment term

$$\boxed{Q \leq t_{\mathsf{i}}'(R_{\mathsf{G}} + Y_{\mathsf{Gseller}}) - (t_{\mathsf{i}} - t_{\mathsf{i}}')Y_{\mathsf{G}} - \frac{l^{n_{\mathsf{R}}}(1 - l)}{1 - l^{n_{\mathsf{R}}}} L_0} \tag{23}$$

This expresses the interests of both the business and the buyer. The buyer and the seller can therefore conclude a deal provided that Q satisfies both Eqs. (13) (the seller's interest) and (23) (the buyer's interest).

What then is the effective net cost to the buyer of following through with the buy-sell transaction? Since the net monetary worth of the buyer is affected through ownership of the business by the distribution of dividends, the effective net cost must equate to the downward change in this distribution. Using Eqs. (21) and (22)

$$D' = t_{\mathsf{ec}} \left[R_{\mathsf{G}} + Y_{\mathsf{Gseller}} - \frac{t_{\mathsf{i}} - t_{\mathsf{i}}'}{t_{\mathsf{i}}'} Y_{\mathsf{G}} - \frac{1}{t_{\mathsf{i}}'}(P + Q) \right]$$

so that

$$\Delta D \equiv D' - D = t_{\mathsf{ec}} \left[Y_{\mathsf{Gseller}} - \frac{t_{\mathsf{i}} - t_{\mathsf{i}}'}{t_{\mathsf{i}}'} Y_{\mathsf{G}} - \frac{1}{t_{\mathsf{i}}'}(P + Q) \right]$$

Obviously, although the total payment made by the buyer is $P_{\mathsf{tot}} = n_{\mathsf{R}}(P + Q)$, the effective total cost to the buyer over the payment term spanning n_{R} months is

$$C_{\mathsf{tot}} = n_{\mathsf{R}} \Delta D$$

6.3 Selling the Business

In the second scenario, the buyer and seller agree that the buyer starts a new business and that the new business purchases the assets and liabilities of the seller's existing business. To be sure, the new business purchases from the existing business, and not from the individual seller. Although the seller's existing business remains as is, it ceases to trade because all trading is done in the buyer's new business.

In the new business, the buyer does not require his or her salary to be increased because it is the new business entity, not the buyer, which is making the payment P to the lender, and the payment Q to the existing business.

The buyer's new business obtains the loan, L_0. CGT is imposed on L_0 before it can be transferred to the seller's existing business. That is $(1 - \tau_{\mathsf{ccgt}})L_0 \equiv t_{\mathsf{ccgt}} L_0$ is transferred. STC is then levied on this amount when the seller uses it to augment his or her initial net monetary worth, W_0. That is, the seller declares a dividend, D_0, as

$$D_0 = t_{\mathsf{ccgt}} L_0 - \tau_{\mathsf{stc}} D_0$$

A simple form is

$$D_0 = \frac{t_{\text{ccgt}}}{t_{\text{stc}}} L_0$$

where, as before (Eq. (20)), $t_{\text{stc}} \equiv 1 + \tau_{\text{stc}}$.

Then, obviously, the seller's initial net monetary worth is

$$W_0 = K_0 + D_0 = K_0 + \frac{t_{\text{ccgt}}}{t_{\text{stc}}} L_0$$

What about the subsequent monthly payments, Q, to the seller's business? Since, as with L_0, the payments from the new business pass through the seller's existing business, the seller must declare the dividend $(t_{\text{ccgt}}/t_{\text{stc}})Q$.

Once again, if we also assume that the seller's monthly income from labour, I_n, during the term is nil, then it is easy to show that the expression for the seller's net monetary worth at month n_{R} becomes

$$W_{n_{\text{R}}} = s^{n_{\text{R}}} \left(K_0 + \frac{t_{\text{ccgt}}}{t_{\text{stc}}} L_0 \right) + \frac{s^{n_{\text{R}}} - 1}{s - 1} \frac{t_{\text{ccgt}}}{t_{\text{stc}}} Q - e \frac{s^{n_{\text{R}}} - e^{n_{\text{R}}}}{s - e} E_0 \tag{24}$$

Imposing the retirement criterion, Eq. (7), at month n_{R} gives a criterion for the payments, Q:

$$\boxed{Q \geq \frac{t_{\text{stc}}}{t_{\text{ccgt}}} \frac{s^{n_{\text{R}}}(s-1)}{s^{n_{\text{R}}} - 1} \left[\frac{e}{s - e} E_0 - K_0 - \frac{t_{\text{ccgt}}}{t_{\text{stc}}} L_0 \right]} \tag{25}$$

This expresses the interests of the seller. And obviously, the total payment received by the seller is

$$Q_{\text{tot}} = \frac{t_{\text{ccgt}}}{t_{\text{stc}}} (L_0 + n_{\text{R}} Q)$$

After the transaction, the buyer's gross salary need not change, i.e.,

$$\boxed{Y_{\text{G}}{}' = Y_{\text{G}}} \tag{26}$$

But, assuming no additional income, the new business's gross profitability, $R_{\text{G}}{}'$, will drop for the duration of the term because the new business will be making the payments P and Q each month. Thus, I may write, as before,

$$\begin{aligned} R_{\text{G}} &= I_{\text{G}} - E_{\text{G}} \\ R_{\text{G}}{}' &= I_{\text{G}} - E_{\text{G}}{}' \end{aligned}$$

Since the new business is making the payments each month, we may partition the new business's monthly expenses as

$$\begin{aligned} E_{\text{G}}{}' &= E_{\text{G}} + P + Q - Y_{\text{Gseller}} \\ &= E_{\text{G}} + P_{\text{capital}} + P_{\text{interest}} + Q - Y_{\text{Gseller}} \end{aligned}$$

Thus,

$$R_{\text{G}}{}' = R_{\text{G}} - (P_{\text{capital}} + P_{\text{interest}} + Q - Y_{\text{Gseller}}) \tag{27}$$

Since the interest payments P_{interest} are tax deductible, the company income tax can be reduced accordingly. So the new business's net profitability is

$$R_{\text{N}}{}' = R_{\text{G}}{}' - (\tau_{\text{c}} R_{\text{G}}{}' - \tau_{\text{c}} P_{\text{interest}})$$

This expresses a reduction in company income tax by an amount equal to the deduction $\tau_{\text{c}} P_{\text{interest}}$. Applying Eq. (27), we obtain an expression for the net profitability in terms of known monthly quantities:

$$\begin{aligned} R_{\text{N}}{}' &= t_{\text{c}} R_{\text{G}}{}' + (1 - t_{\text{c}}) P_{\text{interest}} \\ &= t_{\text{c}} (R_{\text{G}} - P_{\text{capital}} - 2 P_{\text{interest}} - Q + Y_{\text{Gseller}}) + P_{\text{interest}} \end{aligned} \tag{28}$$

where, as in Eq. (19), $t_{\mathrm{c}} \equiv 1 - \tau_{\mathrm{c}}$.

Once again, it is in the interests of the new business to stipulate that

$$\boxed{R_{\mathrm{N}}{}' \geq 0}$$

with which we may place an upper bound on the new business's monthly payments to the seller's business over the payment term, as

$$Q \leq R_{\mathrm{G}} + Y_{\mathrm{Gseller}} - P_{\mathrm{capital}} - \frac{2t_{\mathrm{c}} - 1}{t_{\mathrm{c}}} P_{\mathrm{interest}} \tag{29}$$

But what exactly are P_{capital} and P_{interest} besides the obvious fact that $P_{\mathrm{capital}} + P_{\mathrm{interest}} = P$, the monthly loan repayments? The simplest partition of P is arguably one of equal payments toward reducing the loan capital, and payments toward reducing interest. That is,

$$\begin{aligned} n_{\mathrm{R}} P_{\mathrm{capital}} &= L_0 \\ P_{\mathrm{interest}} &= P - P_{\mathrm{capital}} \end{aligned}$$

so that, using Eq. (11)

$$\begin{aligned} P_{\mathrm{capital}} &= L_0 / n_{\mathrm{R}} \\ P_{\mathrm{interest}} &= \left[\frac{l^{n_{\mathrm{R}}}(1-l)}{1 - l^{n_{\mathrm{R}}}} - \frac{1}{n_{\mathrm{R}}} \right] L_0 \end{aligned} \tag{30}$$

Finally, inserting Eq. (30) into Eq. (29) provides the upper bound on the new business's monthly payments, Q, to the seller's business as

$$\boxed{Q \leq R_{\mathrm{G}} + Y_{\mathrm{Gseller}} - \left[\frac{1}{n_{\mathrm{R}}} + \frac{2t_{\mathrm{c}} - 1}{t_{\mathrm{c}}} \left(\frac{l^{n_{\mathrm{R}}}(1-l)}{1 - l^{n_{\mathrm{R}}}} - \frac{1}{n_{\mathrm{R}}} \right) \right] L_0} \tag{31}$$

This expresses the interests of the business and the buyer. The buyer and the seller therefore have a deal provided that Q satisfies both Eqs. (25) and (31).

As in section 6.2 on page 9, the effective net cost to the buyer of following through with the buy-sell transaction must be the downward change in distributed dividends. Using Eqs. (21), (27) and (30), this change is

$$\Delta D = t_{\mathrm{ec}} (Y_{\mathrm{Gseller}} - P - Q)$$

And, obviously, the total effective net cost is $n_{\mathrm{R}} \Delta D$.

The new business's net profitability may be expressed in terms of the known context of seller's existing business (R_{G} and Y_{Gseller}), the deal parameters (Q and L_0) and taxes (t_{c}). Using Eqs. (28), (30), it is easy to show that

$$\boxed{R_{\mathrm{N}}{}' = t_{\mathrm{c}} \left[R_{\mathrm{G}} + Y_{\mathrm{Gseller}} - Q - \left(\frac{(2 - 1/t_{\mathrm{c}}) l^{n_{\mathrm{R}}}(1-l)}{1 - l^{n_{\mathrm{R}}}} + \frac{1}{n_{\mathrm{R}}} (1/t_{\mathrm{c}} - 1) \right) L_0 \right]} \tag{32}$$

7 Summary

MANY EQUATIONS HAVE BEEN WRITTEN. Let me to summarise. In selling a business, a seller may wish that the proceeds of the sale will ensure that the seller's net monetary worth satisfies some retirement criterion. I choose the criterion Eq. (7) applied to some future month n_{R}. That is, I actually choose Eq. (8). However, because the buyer may not be able to immediately afford to purchase the business, the seller may accept an initial lump sum payment from the buyer, followed by a second payment spread over n_{R} months in equal monthly payments.

Should the buyer and seller agree, then the buy-sell transaction may be done according to one of two scenarios. In the first scenario, the seller's shareholding in the business is sold (Section 6.2). All assets and liabilities in the business remain in the business, and business continues as usual with the buyers being the new owners.

In the second scenario, the seller retains his or her shareholding in the seller's existing business, but the assets and liabilities of the business are sold by the seller to a new business (Section 6.3), and the seller's business ceases trading. The new business "perceives" the seller's business, i.e., not the selling individual, as the seller.

7.1 Selling the Member's Interest in a Business

1. The seller imposes the retirement criterion Eq. (7) applied to the seller's net monetary worth at month n_{R} (Eq. (12) This in turn places a constraint on the monthly payments, Q, made by the buyer to the seller as (Eq. (13))

$$Q \geq \frac{1}{t_{\mathsf{icgt}}} \frac{s^{n_{\mathsf{R}}}(s-1)}{s^{n_{\mathsf{R}}}-1} \left[\frac{e}{s-e} E_0 - K_0 - t_{\mathsf{icgt}} L_0 \right]$$

2. The buyer obtains the loan, L_0, from a third-party lender. In servicing the loan, the buyer must make monthly payments, P, back to the lender for a period spanning n_{R} months, according to (Eq. (11))

$$P = \frac{l^{n_{\mathsf{R}}}(1-l)}{1-l^{n_{\mathsf{R}}}} L_0$$

3. But can the buyer afford to make the above monthly payments P and Q? Yes, provided that after the transaction the business can assist the buyer by increasing the buyer's gross monthly salary, Y_{G}, over n_{R} months to (Eq. (17))

$$Y_{\mathsf{G}}' = \frac{t_{\mathsf{i}}}{t_{\mathsf{i}}'} Y_{\mathsf{G}} + \frac{1}{t_{\mathsf{i}}'}(P+Q)$$

4. But can the business assist in this manner? Yes, provided the monthly payments to the seller are small enough to satisfy (Eq. (23))

$$Q \leq t_{\mathsf{i}}'(R_{\mathsf{G}} + Y_{\mathsf{Gseller}}) - (t_{\mathsf{i}} - t_{\mathsf{i}}')Y_{\mathsf{G}} - \frac{l^{n_{\mathsf{R}}}(1-l)}{1-l^{n_{\mathsf{R}}}} L_0$$

   A detailed analysis of the business's current gross monthly income and expenses (but after VAT has been paid) affords an answer to this.

5. Once Q has been agreed upon, the business's new net profitability is (Eq. (22))

$$R_{\mathsf{N}}' = t_{\mathsf{c}} \left[R_{\mathsf{G}} + Y_{\mathsf{Gseller}} - \frac{t_{\mathsf{i}} - t_{\mathsf{i}}'}{t_{\mathsf{i}}'} Y_{\mathsf{G}} - \frac{1}{t_{\mathsf{i}}'} Q - \frac{1}{t_{\mathsf{i}}'} \frac{l^{n_{\mathsf{R}}}(1-l)}{1-l^{n_{\mathsf{R}}}} L_0 \right]$$

7.2 Selling the Business

1. The seller imposes the retirement criterion Eq. (7) applied to the seller's net monetary worth at month n_{R} (Eq. (24) This in turn places a constraint on the monthly payments, Q, made by the buyer's new business to the seller's business as (Eq. (25))

$$Q \geq \frac{t_{\mathsf{stc}}}{t_{\mathsf{ccgt}}} \frac{s^{n_{\mathsf{R}}}(s-1)}{s^{n_{\mathsf{R}}}-1} \left[\frac{e}{s-e} E_0 - K_0 - \frac{t_{\mathsf{ccgt}}}{t_{\mathsf{stc}}} L_0 \right]$$

2. The buyer's new business obtains the loan, L_0, from a third-party lender. In servicing the loan, the buyer's business must make monthly payments, P, back to the lender for a period spanning n_{R} months, according to (Eq. (11))

$$P = \frac{l^{n_{\mathsf{R}}}(1-l)}{1-l^{n_{\mathsf{R}}}} L_0$$

3. The buyer's business continues to pay the buyer individual a gross monthly salary equal to Y_G, i.e. Eq. (26) applies:

$$Y_\mathrm{G}' = Y_\mathrm{G}$$

4. But can the buyer's new business maintain the payments, P and Q? Yes, provided the monthly payments to the seller's business are small enough to satisfy (Eq. (31))

$$Q \leq R_\mathrm{G} + Y_\mathrm{Gseller} - \left[\frac{1}{n_\mathrm{R}} + \frac{2t_\mathrm{c} - 1}{t_\mathrm{c}}\left(\frac{l^{n_\mathrm{R}}(1-l)}{1-l^{n_\mathrm{R}}} - \frac{1}{n_\mathrm{R}}\right)\right] L_0$$

A detailed analysis of the business's current gross monthly income and expenses (but after VAT has been paid) will show whether or not this is satisfied.

5. Once Q has been agreed upon, the new business's net profitability is (Eq. (32))

$$R_\mathrm{N}' = t_\mathrm{c}\left[R_\mathrm{G} + Y_\mathrm{Gseller} - Q - \left(\frac{(2-1/t_\mathrm{c})l^{n_\mathrm{R}}(1-l)}{1-l^{n_\mathrm{R}}} + \frac{1}{n_\mathrm{R}}(1/t_\mathrm{c} - 1)\right) L_0\right]$$

8 Buy-Sell Transaction Scenarios

By varying the various input parameters in the above analyses, the two broad scenarios modelled in Section 6 and summarised in Section 7 may be used to model specific buy-sell transaction scenarios. These are:

1. Initial lump sum payment to seller, followed by fixed monthly payments. Lender becomes part shareholder. Seller's shareholding in business sold to individual buyer(s).
2. Zero lump sum, only monthly payments. Seller's shareholding in business sold to individual buyer(s).
3. Initial lump sum payment to seller, followed by fixed monthly payments. Lender becomes part shareholder. Assets and liabilities sold out of seller's business to a new business.
4. Zero lump sum, only monthly payments. Assets and liabilities sold out of seller's business to a new business.
5. Zero lump sum, only monthly payments. Actual transaction delayed until seller's initial net worth reaches threshold. Meantime, seller takes dividends. Buyer(s) get full management control now (just not yet financial).
6. As per Scenario 1, except seller equals the lender. Seller retains a possibly nil shareholding.
7. As per Scenario 3, except seller equals the lender. Seller retains a possibly nil shareholding.
8. As per Scenario 3, except seller equals the lender. Seller retains a possibly nil shareholding.
9. Full initial lump sum payment to seller with zero following payments. Lender becomes part shareholder. Seller's shareholding in business sold to individual buyer(s).
10. Full initial lump sum payment to seller with zero following payments. Lender becomes part shareholder. Assets and liabilities sold out of seller's business to a new business.
11. As per Scenarios 1–8, except seller is given a labour-free salary.